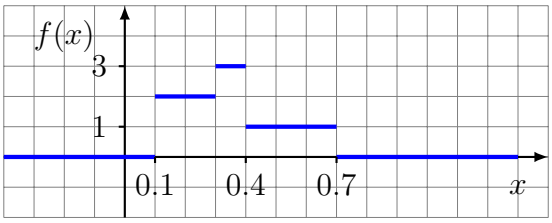
Aufgabe 1

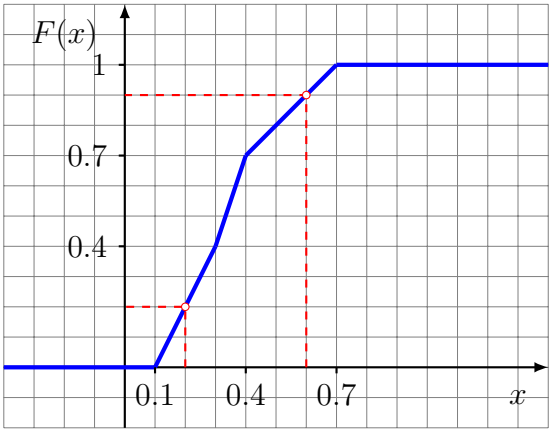
$$f(x) = \begin{cases} 2 & \text{wenn } 0.1 \leq x < 0.3 \\ 3 & \text{wenn } 0.3 \leq x < 0.4 \\ 1 & \text{wenn } 0.4 \leq x < 0.7 \\ 0 & \text{sonst} \end{cases}$$

(a)
- $f(x) \geq 0$ für alle $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} f(x)\,\mathrm{d}x = 0.2 \cdot 2 + 0.1 \cdot 3 + 0.3 \cdot 1 = 1$

(b) Graph der Wahrscheinlichkeitsdichte f

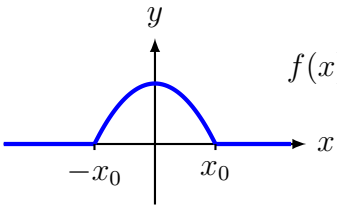


(c) Graph der Verteilungsfunktion F von f.



(d) $P(0.2 \leq X \leq 0.6) = F(0.6) - F(0.2) = 0.9 - 0.2 = 0.7$

Aufgabe 2



$$f(x) = \begin{cases} \frac{3}{2} - ax^2 & \text{für } -x_0 \le x \le x_0 \\ 0 & \text{sonst} \end{cases}$$

(a) $f(x_0) = 0 \quad \Rightarrow \quad \frac{3}{2} - ax_0^2 = 0 \quad \Rightarrow \quad x_0 = \sqrt{\frac{3}{2a}}$

$$\int_0^{x_0} \left(\tfrac{3}{2} - ax^2\right) \mathrm{d}x = \tfrac{1}{2} \quad \text{(Symmetrie von } f\text{)}$$

$$\left[\tfrac{3}{2}x - \tfrac{1}{3}ax^3\right]_0^{x_0} = \tfrac{1}{2}$$

$$\tfrac{3}{2}\sqrt{\tfrac{3}{2a}} - \tfrac{a}{3} \cdot \tfrac{3}{2a}\sqrt{\tfrac{3}{2a}} = \tfrac{1}{2}$$

$$\sqrt{\tfrac{3}{2a}} = \tfrac{1}{2} \quad \Rightarrow \quad a = 6 \quad \Rightarrow \quad x_0 = \tfrac{1}{2}$$

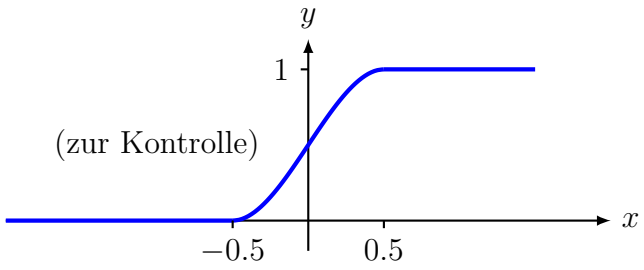
(b) $$f(x) = \begin{cases} \frac{3}{2} - 6x^2 & \text{für } -0.5 \le x \le 0.5 \\ 0 & \text{sonst} \end{cases}$$

$F(x)$ erhält man durch Integration aus $f(x)$:

$$F(x) = \int_{-0.5}^{x} \left(\tfrac{3}{2} - 6t^2\right) \mathrm{d}t = \left[\tfrac{3}{2}t - 2t^3\right]_{-0.5}^{x} = \tfrac{3}{2}x - 2x^3 + 0.5$$

$$F(x) = \begin{cases} 0 & \text{für } x < -0.5 \\ \frac{3}{2}x - 2x^3 + 0.5 & \text{für } -0.5 \le x < 0.5 \\ 1 & 0.5 \le x \end{cases}$$

(zur Kontrolle)



(c) $P(-0.2 \le X \le 0.3) = F(0.3) - F(-0.2) = 0.896 - 0.216 = 0.68$