# Analysis (Aufgabenheft)

## Kapitel 4

# Aufgabe 4.4

(c) $\displaystyle\int_1^4 x^5 (2x+5)\,\mathrm{d}x$

(d) $\displaystyle\int_{-2}^2 (2x-3)^2\,\mathrm{d}x$

# Aufgabe 4.4

(a) $$\int_{-3}^{-2} \left(x^3 + 6x\right) \mathrm{d}x = \left[\tfrac{1}{4}x^4 + 3x^2\right]_{-3}^{-2}$$

$$= 16 - \frac{189}{4} = -\frac{125}{4} = -31.25$$

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(b) $$\int_{-2}^{1} \left(x^4 - 5\right) \mathrm{d}x = \left[\tfrac{1}{5}x^5 - 5x\right]_{-2}^{1}$$

$$= -\tfrac{24}{5} - \tfrac{18}{5} = -\tfrac{42}{5} = -8.4$$

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(c) $$\int_{1}^{4} x^5(2x + 5)\,\mathrm{d}x = \int_{1}^{4} \left(2x^6 + 5x^5\right) \mathrm{d}x = \left[\tfrac{2}{7}x^7 + \tfrac{5}{6}x^6\right]_{1}^{4}$$

$$\approx 8093.56$$

(d) $$\int_{-2}^{2}(2x-3)^2\,\mathrm{d}x = \int_{-2}^{2}(4x^2-12x+9)\,\mathrm{d}x$$

$$= \left[\tfrac{4}{3}x^3-6x^2+9x\right]_{-2}^{2}$$

$$= \tfrac{14}{3}-\left(-\tfrac{158}{3}\right)=\tfrac{172}{3}=57.\overline{3}$$

# Aufgabe 4.5

Ermittle $k$ für

(a) $\displaystyle\int_1^3 \left(1.5x^2 + 3x + k\right) \mathrm{d}x = 17$

(b) $\displaystyle\int_{-1}^0 \left(3x^2 - kx + k\right) \mathrm{d}x = -2$

# Aufgabe 4.5

(a)

$$\begin{aligned}
\int_1^3 \left(1.5x^2 + 3x + k\right) \mathrm{d}x &= 17 \\
\left[\tfrac{1}{2}x^3 + \tfrac{3}{2}x^2 + kx\right]_1^3 &= 17 \\
\left(\tfrac{27}{2} + \tfrac{27}{2} + 3k\right) - \left(\tfrac{1}{2} + \tfrac{3}{2} + k\right) &= 17 \\
25 + 2k &= 17 \\
k &= -4
\end{aligned}$$

(b) $\int_{-1}^{0} \left(3x^2 - kx + k\right) \mathrm{d}x = -2$

$$\left[x^3 - \tfrac{k}{2}x^2 + kx\right]_{-1}^{0} = -2$$

$$0 - \left( - 1 - \tfrac{k}{2} - k\right) = -2$$

$$1 + \tfrac{3k}{2} = -2$$

$$\tfrac{3k}{2} = -3$$

$$3k = -6$$

$$k = -2$$

# Aufgabe 4.6

Berechne den Inhalt der vom Graphen der Funktion $f: x \mapsto y$ und der $x$-Achse eingeschlossenen Flächenstücke.

(a) $f: y = x^2 - 5x + 4$

(b) $f: y = x^3 - 2x^2 - 3x$

(c) $f: y = x^4 - 10x^2 + 9$

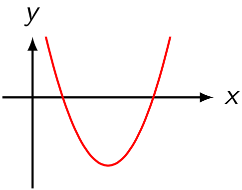
# Aufgabe 4.6

(a) $f(x) = x^2 - 5x + 4 = (x-1)(x-4)$

Nullstellen: $x_1 = 1,\ x_2 = 4$

asymptotisches Verhalten: $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x^2 = +\infty$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^2 = +\infty$$




$$I = \int_1^4 (x^2 - 5x + 4)\,\mathrm{d}x = \left[\tfrac{1}{3}x^3 - \tfrac{5}{2}x^2 + 4x\right]_1^4 = -4.5$$

# Aufgabe 4.6

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Nullstellen: $x_1 = 1,\ x_2 = 4$

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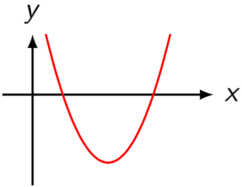


$$I = \int_1^4 (x^2 - 5x + 4)\,\mathrm{d}x = \left[\tfrac{1}{3}x^3 - \tfrac{5}{2}x^2 + 4x\right]_1^4 = -4.5$$

$A = |I| = 4.5$

(b) $f(x) = x^3 - 2x^2 - 3x = x(x^2 - 2x - 3) = x(x-3)(x+1)$

Nullstellen: $x_1 = -1$, $x_2 = 0$, $x_3 = 3$

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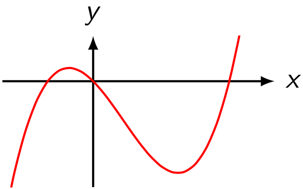


$$I_1 = \int_{-1}^{0} f(x)\,\mathrm{d}x = \left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - \tfrac{3}{2}x^2\right]_{-1}^{0} = \tfrac{7}{12}$$

$$I_2 = \int_{0}^{3} f(x)\,\mathrm{d}x = \left[\ldots\right]_{0}^{3} = -\tfrac{45}{4}$$

(b) $f(x) = x^3 - 2x^2 - 3x = x(x^2 - 2x - 3) = x(x-3)(x+1)$

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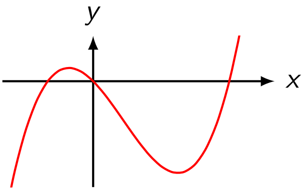


$$I_1 = \int_{-1}^{0} f(x)\,\mathrm{d}x = \left[\tfrac{1}{4}x^4 - \tfrac{2}{3}x^3 - \tfrac{3}{2}x^2\right]_{-1}^{0} = \tfrac{7}{12}$$

$$I_2 = \int_{0}^{3} f(x)\,\mathrm{d}x = \left[\ldots\right]_{0}^{3} = -\tfrac{45}{4}$$

$$A = |I_1| + |I_2| = \tfrac{71}{6} = 11.8\overline{3}$$

(c) $f(x) = x^4 - 10x^2 + 9 = (x^2 - 1)(x^2 - 9)$

$$= (x-1)(x+1)(x-9)(x+9)$$

Nullstellen: $x_1 = -3,\ x_2 = -1,\ x_3 = 1,\ x_4 = 3$

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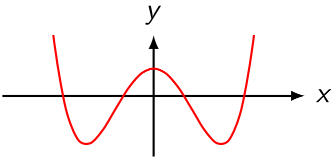


$$I_1 = \int_0^1 f(x)\,\mathrm{d}x = \left[\tfrac{1}{5}x^5 - \tfrac{10}{3}x^3 + \tfrac{9}{2}x\right]_0^1 = \tfrac{88}{15}$$

$$I_2 = \int_1^3 f(x)\,\mathrm{d}x = \left[\ \dots\ \right]_1^3 = -\tfrac{304}{15}$$

(c) $f(x) = x^4 - 10x^2 + 9 = (x^2 - 1)(x^2 - 9)$
$= (x - 1)(x + 1)(x - 9)(x + 9)$

Nullstellen: $x_1 = -3,\ x_2 = -1,\ x_3 = 1,\ x_4 = 3$

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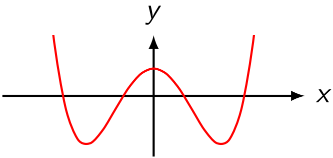


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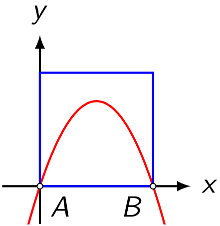
$$A = 2(|I_1| + |I_2|) = \tfrac{784}{15} = 52.2\overline{6} \qquad \text{(wegen Symmetrie)}$$

# Aufgabe 4.7

Die Parabel $p\colon y = 3x - x^2$ schneidet die $x$-Achse in den Nullstellen $A$ und $B$. Zeige, dass die Parabel das im 1. Quadranten liegende Quadrat mit der Seite $AB$ halbiert.

# Aufgabe 4.7

$$p(x) = 3x - x^2 = x(3 - x) \quad \Rightarrow \quad x_1 = 0,\ x_2 = 3$$




Test: $f(1.5) = 1.5(3 - 1.5) = 2.25 \leq 3$ (ok)

$$A_p = \int_0^3 f(x)\,\mathrm{d}x = \left[\tfrac{3}{2}x^2 - \tfrac{1}{3}x^3\right]_0^3 = 13.5 - 9 - (0 - 0) = 4.5$$

# Aufgabe 4.7

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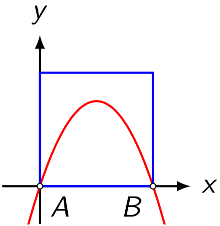


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$\frac{1}{2}\overline{AB}^2 = \frac{1}{2} \cdot 9 = 4.5$ (ok)

# Aufgabe 4.8

Die Parabel $p\colon y = ax - x^3$ schliesst im 1. Quadranten mit der $x$-Achse eine Fläche vom Inhalt $A = 9$ ein. Bestimme den Wert von $a$.

# Aufgabe 4.8

$$p(x) = ax - x^3 = x(a - x^2) = x\left(\sqrt{a} - x\right)\left(\sqrt{a} + x\right)$$

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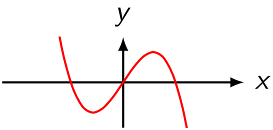

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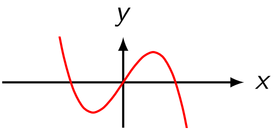


$$\int_0^{\sqrt{a}} \left(ax - x^3\right) \mathrm{d}x = 9$$

$$\left[\tfrac{1}{2}ax^2 - \tfrac{1}{4}x^4\right]_0^{\sqrt{a}} = 9$$

$$\tfrac{1}{2}a^2 - \tfrac{1}{4}a^2 = 9$$

$$\tfrac{1}{4}a^2 = 9$$

$$a = 6$$

# Aufgabe 4.9

Eine Parabel 3. Ordnung hat in $P(1, 0)$ die Steigung $m = 1$ und berührt die $x$-Achse im Koordinatenursprung.

(a) Bestimme die Parabelgleichung, skizziere die Parabel.

(b) Berechne den Inhalt der Fläche zwischen der Parabel und der $x$-Achse.

# Aufgabe 4.9

Ansatz: $f(x) = ax^3 + bx^2 + cx + d$

$$f'(x) = 3ax^2 + 2bx + c$$

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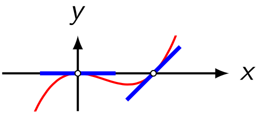

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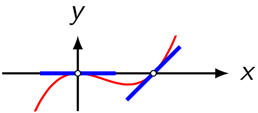


$f(1) = 0$: $\quad a + b + c + d = 0 \quad (1)$

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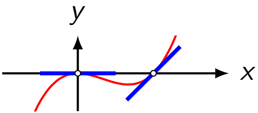


$f(1) = 0$: $\quad a + b + c + d = 0 \quad (1)$

$f'(1) = 1$: $\quad 3a + 2b + c = 1 \quad (2)$

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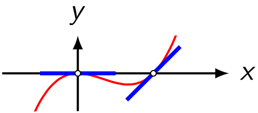


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$f(0) = 0$: $\quad d = 0 \quad (3)$

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Ansatz: $f(x) = ax^3 + bx^2 + cx + d$

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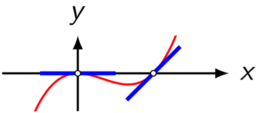


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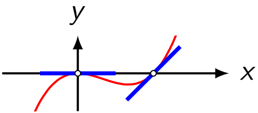


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$f(0) = 0$: $\quad d = 0 \quad (3)$

$f'(0) = 0$: $\quad c = 0 \quad (4)$

$$\begin{array}{r} a + b = 0 \\ 3a + 2b = 1 \end{array} \quad \Rightarrow \quad \begin{array}{l} a = 1 \\ b = -1 \end{array} \quad \Rightarrow \quad f(x) = x^3 - x^2$$

$$\int_0^1 f(x)\,\mathrm{d}x = \left[\tfrac{1}{4}x^4 - \tfrac{1}{3}x^3\right]_0^1 = -\tfrac{1}{12} \quad \Rightarrow \quad A = \tfrac{1}{12}$$

# Aufgabe 4.13

(a) Bestimme $a$ so, dass die Parabel $p\colon\, y = 2x^3 - 6x + a$ die $x$-Achse in einem Tiefpunkt berührt.

(b) Skizziere die Parabel und berechne den Inhalt zwischen Parabel und $x$-Achse.

# Aufgabe 4.13

$$p(x) = 2x^3 - 6x + a$$
$$p'(x) = 6x^2 - 6$$
$$p''(x) = 12x$$

(a) Tiefstellen:

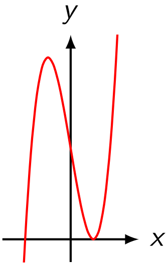
$$p'(x) = 0$$
$$6x^2 - 6 = 0$$
$$6(x^2 - 1) = 0$$
$$x_1 = -1 \;\Rightarrow\; p''(-1) = -12 < 0 \;\Rightarrow\; \text{HoP}(-1, 4 + a)$$
$$x_2 = 1 \;\Rightarrow\; p''(1) = 12 > 0 \;\Rightarrow\; \text{TiP}(1, -4 + a)$$

Die Parabel $p$ soll die $x$-Achse in $\text{TiP}(1, -4 + a)$ berühren.

$$\Rightarrow a = 4 \quad \Rightarrow \quad p(x) = 2x^3 - 6x + 4$$

(b) $p(x) = 2x^3 - 6x + 4$

Nullstellen: $x_1 = x_2 = 1$, $x_3 = -2$; Ordinatenabschnitt: $y_0 = 4$




$$A = \int_{-2}^{1} \left(2x^3 - 6x + 4\right) \mathrm{d}x = \left[\tfrac{1}{2}x^4 - 3x^2 + 4x\right]_{-2}^{1} = 13.5$$

# Aufgabe 4.14

Berechne den Inhalt des Flächenstückes, das die Graphen der Funktionen $f_1$ und $f_2$ einschliessen.

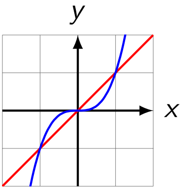
(b) $f_1 : y = x$ $\qquad$ $f_2 : y = x^3$

(c) $f_1 : y = 2x - 3$ $\qquad$ $f_2 : y = x^2 - 2x - 8$

(d) $f_1 : y = x^3$ $\qquad$ $f_2 : y = 2x - x^2$

# Aufgabe 4.14

(b) $f_1\colon y = x$ $\qquad$ $f_2\colon y = x^3$



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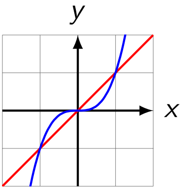



$$A = 2\int_0^1 \left(x - x^3\right) = 2\left[\tfrac{1}{2}x^2 - \tfrac{1}{4}x^4\right]_0^1 = 2\cdot\left(\tfrac{1}{2} - \tfrac{1}{4}\right) = \tfrac{1}{2}$$

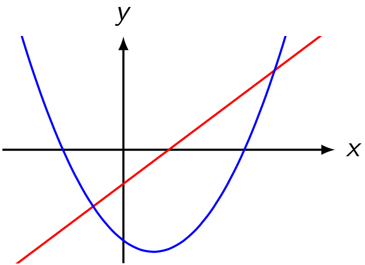
(c) $f_1 \colon y = 2x - 3 \qquad f_2 \colon y = x^2 - 2x - 8$

$$2x - 3 = x^2 - 2x - 8$$

$$0 = x^2 - 4x - 5 = (x-5)(x+1)$$

$$x_1 = -1 \quad \Rightarrow \quad y_1 = -5$$

$$x_2 = 5 \quad \Rightarrow \quad y_2 = 7$$



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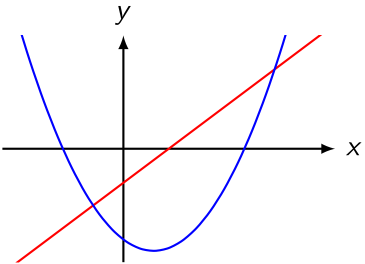



$$A = \int_{-1}^{5} \big(2x - 3 - \big(x^2 - 2x - 8\big)\big)\, \mathrm{d}x$$

$$= \int_{-1}^{5} \big(-x^2 + 4x + 5\big)\, \mathrm{d}x = \Big[-\tfrac{1}{3}x^3 + x^2 + 5x\Big]_{-1}^{5} = 36$$

(d) $f_1\colon y = x^3$ $\qquad$ $f_2\colon y = 2x - x^2$

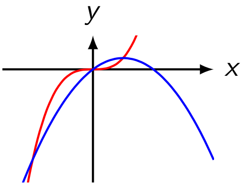
$$x^3 = 2x - x^2$$

$$0 = x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x-1)(x+2)$$

$$x_1 = 0 \quad \Rightarrow \quad y_1 = 0$$

$$x_2 = 1 \quad \Rightarrow \quad y_2 = 1$$

$$x_3 = -2 \quad \Rightarrow \quad y_3 = -8$$



(d) $f_1\colon y = x^3$ $\qquad f_2\colon y = 2x - x^2$

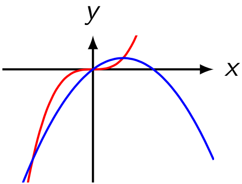
$$x^3 = 2x - x^2$$

$$0 = x^3 + x^2 - 2x = x(x^2 + x - 2) = x(x-1)(x+2)$$

$$x_1 = 0 \quad \Rightarrow \quad y_1 = 0$$

$$x_2 = 1 \quad \Rightarrow \quad y_2 = 1$$

$$x_3 = -2 \quad \Rightarrow \quad y_3 = -8$$




$$A_1 = \int_{-2}^{0} \left(x^3 + x^2 - 2x\right) \mathrm{d}x \quad = \left[\tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 - x^2\right]_{-2}^{0} = \frac{8}{3}$$

$$A_2 = \int_{1}^{0} \left(x^3 + x^2 - 2x\right) \mathrm{d}x \quad = \left[\tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 - x^2\right]_{1}^{0} = \frac{5}{12}$$

$$A = A_1 + A_2 = \frac{37}{12}$$

# Aufgabe 4.15

(a) Bestimme die Extrema der Parabel $p\colon y = x^3 - 3x^2 + 6$ und skizziere die Parabel.

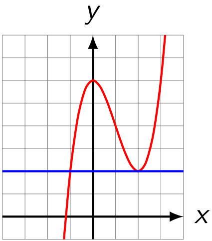
(b) Welchen Inhalt hat die Fläche zwischen Parabel und Tangente im Tiefpunkt?

# Aufgabe 4.15

$$p(x) = x^3 - 3x^2 + 6$$
$$p'(x) = 3x^2 - 6x = 3x(x-2)$$
$$p''(x) = 6x - 6 = 6(x-1)$$

(a) $p'(x) = 0$

$$x_1 = 0 \quad \Rightarrow \quad p''(0) = -6 < 0 \quad \Rightarrow \quad \text{HoP}(0,6)$$
$$x_2 = 2 \quad \Rightarrow \quad p''(2) = 6 > 0 \quad \Rightarrow \quad \text{TiP}(2,2)$$



(b) $t: y = 2$ (parallel zur $x$-Achse durch TiP)

$$p(x) = t(x)$$
$$x^3 - 3x^2 + 6 = 2$$
$$x^3 - 3x^2 + 4 = 0$$
$$x_1 = x_2 = 2$$
$$x_3 = -1$$

$$A = \int_{-1}^{2} \left(f(x) - t(x)\right) \mathrm{d}x = \int_{-1}^{2} \left(x^3 - 3x^2 + 4\right) \mathrm{d}x$$

$$= \left[\tfrac{1}{4}x^4 - x^3 + 4x\right]_{-1}^{2} = \tfrac{27}{4} = 6.75$$

# Aufgabe 4.19

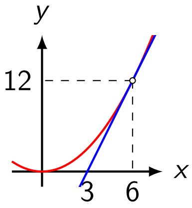
Welchen Inhalt hat die Fläche, die von der Parabel $p\colon x^2 - 3y = 0$, der Kurventangente in $P(6, y)$ und der $x$-Achse begrenzt wird?

# Aufgabe 4.19

$p(x) = \frac{1}{3}x^2$; $p'(x) = \frac{2}{3}x$

Kurventangente in $P(6, y)$ (z. B. via Taylorpolynom)

$t(x) = f(6) + f'(6)(x - 6) = 12 + 4(x - 6) = 4x - 12$




Nullstelle von $t$: $x = 3$

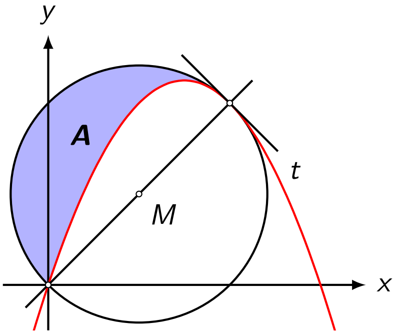
$$A = A_{\text{Parabel}} - A_{\text{Dreieck}} = \int_0^6 \frac{1}{3}x^2 \, dx - \frac{1}{2} \cdot 3 \cdot 12 -$$

$$= \left[\frac{1}{9}x^3\right]_0^6 - 18 = 24 - 18 = 6$$

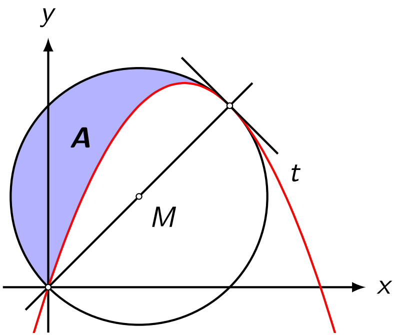
# Aufgabe 4.38

Ein Kreis mit dem Mittelpunkt $M(2,2)$ und eine quadratische Parabel $p$ sind gegeben (siehe Bild). Im Punkt $P$ besitzten sie eine gemeinsame Tangente $t$.

Bestimme die Gleichung der Parabel $p$ und den Inhalt der hervorgehobenen Fläche $A$.



# Aufgabe 4.38




Kreis mit $M(2,2)$ und $r = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

Ansatz: $f(x) = ax^2 + bx \quad (c = 0 \text{ da } (0,0) \in G_f)$

$$f'(x) = 2ax + b$$

$f(4) = 4$: $16a + 4b = 4 \quad \Rightarrow \quad 4a + b = 1 \quad (1)$

$f'(4) = -1$: $8a + b = -1 \quad (2)$

(1) (2) $\Rightarrow \quad f(x) = -\frac{1}{2}x^2 + 3x$

Gleichung des Durchmessers: $g(x) = x$

Fläche zwischen Diagonale und Parabel:

$$I = \int_0^4 \left(f(x) - g(x)\right) \mathrm{d}x = \int_0^4 \left(-\tfrac{1}{2}x^2 + 3x - x\right) \mathrm{d}x$$

$$= \int_0^4 \left(-\tfrac{1}{2}x^2 + 2x\right) \mathrm{d}x = \left[-\tfrac{1}{6}x^3 - x^2\right]_0^4 = \tfrac{16}{3}$$

$A = \frac{1}{2}A_K - I = 4\pi - \frac{16}{3} \approx 7.23$