Aufgabe 1 (1)

$$\int_0^1 \sqrt{3x+1}\,\mathrm{d}x = \dots$$

Substitution: $u = 3x+1 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = 3 \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{3}\,\mathrm{d}u$

$$\dots = \int_1^4 \sqrt{u}\cdot\frac{1}{3}\,\mathrm{d}u = \frac{1}{3}\int_1^4 \sqrt{u}\,\mathrm{d}u = \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4 = \frac{1}{3}\left[\frac{16}{3}-\frac{2}{3}\right] = \frac{14}{9}$$

Aufgabe 2 (4)

$$\int_{-2}^0 \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$$

Substitution: $u = -\frac{1}{2}x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = -\frac{1}{2} \quad \Rightarrow \quad \mathrm{d}x = -2\cdot\mathrm{d}u$

Grenzen: $u(-2) = 1$, $u(0) = 0$

$$\dots = \int_1^0 \mathrm{e}^u\cdot(-2)\cdot\mathrm{d}u = (-2)\int_1^0 \mathrm{e}^u\mathrm{d}u$$

$$= (-2)\left[\mathrm{e}^u\right]_1^0 = -2(1-\mathrm{e}) = 2\mathrm{e}-2$$

Aufgabe 3 (6)

$$\int_0^2 \left(\tfrac{1}{2}x-1\right)^5\mathrm{d}x = \dots$$

Substitution: $u = \frac{1}{2}x-1 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2} \quad \Rightarrow \quad \mathrm{d}x = 2\,\mathrm{d}u$

$$\dots = \int_{-1}^0 u^5\cdot 2\,\mathrm{d}u = 2\left[\frac{1}{6}u^6\right]_{-1}^0 = 2\left(0-\frac{1}{6}\right) = -\frac{1}{3}$$

Aufgabe 4 (10)

$$\int_0^3 \frac{1}{\sqrt{t+1}}\,\mathrm{d}t = \dots$$

Substitution: $u(t) = t+1 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}t} = 1 \quad \Rightarrow \quad \mathrm{d}t = \mathrm{d}u$

$$\dots = \int_1^4 \frac{1}{\sqrt{u}}\,\mathrm{d}u = 2\int_1^4 \frac{1}{2\sqrt{u}}\,\mathrm{d}u = 2\left[\sqrt{u}\right]_1^4 = 2(2-1) = 2$$

Aufgabe 5 (2)

$$\int x\,\mathrm{e}^{x^2}\,\mathrm{d}x = \dots$$

Substitution: $u = x^2 \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2x \quad \Rightarrow \quad \mathrm{d}x = \dfrac{1}{2x}\mathrm{d}u$

$$\dots = \int x\,\mathrm{e}^u \cdot \frac{1}{2x}\mathrm{d}u = \frac{1}{2}\int \mathrm{e}^u \mathrm{d}u = \frac{1}{2}\,\mathrm{e}^u = \frac{1}{2}\,\mathrm{e}^{(x^2)} + C$$

Aufgabe 6 (3)

$$\int \frac{4 \cdot \ln x}{x}\,\mathrm{d}x = \dots$$

Substitution: $u = \ln|x| \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x} \quad \Rightarrow \quad \mathrm{d}x = x \cdot \mathrm{d}u$

$$\dots = 4\int \frac{u}{x} \cdot x\,\mathrm{d}u = 4\int u\,\mathrm{d}u = 4 \cdot \frac{1}{2}\,u^2 = 2(\ln|x|)^2 + C$$

Aufgabe 7 (7)

$$\int x^2\,\sin(x^3)\,\mathrm{d}x = \dots$$

Substitution: $u = x^3 \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 3x^2 \quad \Rightarrow \quad \mathrm{d}x = \dfrac{1}{3x^2}\mathrm{d}u$

$$\dots = \int x^2\,\sin(u) \cdot \frac{1}{3x^2}\,\mathrm{d}u = \frac{1}{3}\int \sin(u)\,\mathrm{d}u = -\frac{1}{3}\cos u = -\frac{1}{3}\cos(x^3) + C$$

Aufgabe 8 (8)

$$\int_0^1 \frac{2x}{x^2+1}\,\mathrm{d}x = \dots$$

Substitution: $u = x^2 + 1 \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2x \quad \Rightarrow \quad \mathrm{d}x = \dfrac{1}{2x}\,\mathrm{d}u$

$$\dots = \int_1^2 \frac{2x}{u} \cdot \frac{1}{2x}\,\mathrm{d}u = \int_1^2 \frac{1}{u}\,\mathrm{d}u = \big[\ln|u|\big]_1^2 = \ln(2) - \ln(1) = \ln(2)$$

Aufgabe 9 (12)

$$\int_0^1 x\ln(x^2+1)\,\mathrm{d}x = \dots$$

Substitution: $u(x) = x^2 + 1 \quad \Rightarrow \quad \mathrm{d}u = 2x\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \dfrac{1}{2x}\,\mathrm{d}u$

$$\ldots = \int_1^2 x \ln|u| \cdot \frac{1}{2x}\,\mathrm{d}u = \frac{1}{2}\int_1^2 \ln|u|\,\mathrm{d}u = \frac{1}{2}\big[u\big(\ln|u| - 1\big)\big]_1^2$$
$$= \frac{1}{2}\big[2(\ln(2)-1) - (\ln(1)-1)\big] = \frac{1}{2}(2\ln(2)-1) = \ln(2) - \frac{1}{2}$$

Aufgabe 10 (14)

$$\int \frac{\mathrm{e}^x}{\mathrm{e}^x - 1}\,\mathrm{d}x = \ldots$$

Substitution: $u(x) = \mathrm{e}^x - 1 \quad \Rightarrow \quad \mathrm{d}u = \mathrm{e}^x\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \mathrm{e}^{-x}\,\mathrm{d}u$

$$\ldots = \int \frac{\mathrm{e}^x}{u} \cdot \mathrm{e}^{-x}\,\mathrm{d}u = \int \frac{1}{u}\,\mathrm{d}u$$
$$= \ln|u| + C_u = \ln|\mathrm{e}^x - 1| + C$$

Aufgabe 11 (15)

$$\int \frac{\ln\left(\sqrt{x}+1\right)}{\sqrt{x}}\,\mathrm{d}x = \ldots$$

Substitution: $u(x) = \sqrt{x} + 1 \quad \Rightarrow \quad \mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = 2\sqrt{x}\,\mathrm{d}u$

$$\ldots = \int \frac{\ln(u)}{\sqrt{x}} \cdot 2\sqrt{x}\,\mathrm{d}u = 2\int \ln(u)\,\mathrm{d}u$$
$$= 2u(\ln|u| - 1) + C_u = 2\big(\sqrt{x}+1\big)\big(\ln\big|\sqrt{x}+1\big| - 1\big) + C$$

Aufgabe 12 (16)

$$\int_{\pi/2}^{\pi} \cos^2 x \sin x\,\mathrm{d}x = \ldots$$

Substitution: $u(x) = \cos x \quad \Rightarrow \quad \mathrm{d}u = -\sin x\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{-1}{\sin x}\,\mathrm{d}u$

$$\ldots = \int u^2 \sin x \cdot \frac{-1}{\sin x}\,\mathrm{d}u = -\int u^2\,\mathrm{d}u = -\tfrac{1}{3}\cos^3 x + C$$

$$\int_{\pi/2}^{\pi} \cos^2 x \sin x\,\mathrm{d}x = \tfrac{1}{3}\big[\cos^3 x\big]_{\pi}^{\pi/2} = \tfrac{1}{3}\big(\cos^3 \tfrac{\pi}{2} - \cos^3 \pi\big) = \tfrac{1}{3}$$

Aufgabe 13 (17) ($\star$)

$$\int \frac{x}{1+x^4}\,\mathrm{d}x = \ldots$$

Substitution: $u(x) = x^2 \quad \Rightarrow \quad \mathrm{d}u = 2x\,\mathrm{d}x \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{2x}\,\mathrm{d}u$

$$\ldots = \int \frac{x}{1+u^2} \cdot \frac{1}{2x}\,\mathrm{d}u = \frac{1}{2}\int \frac{1}{1+u^2}\,\mathrm{d}u$$
$$= \frac{1}{2}\arctan u = \frac{1}{2}\arctan x^2 + C$$

Aufgabe 14 (18) ($\star$)

$$\int \frac{3+2t}{5+2t}\,\mathrm{d}t = \int \frac{(5+2t)-2}{5+2t}\,\mathrm{d}t = \ldots$$

Substitution: $u(t) = 5+2t \quad \Rightarrow \quad \mathrm{d}u = 2\,\mathrm{d}t \quad \Rightarrow \quad \mathrm{d}t = \frac{1}{2}\,\mathrm{d}u$

$$\ldots = \frac{1}{2}\int \frac{u-2}{u}\,\mathrm{d}u = \frac{1}{2}\int \left(1 - \frac{2}{u}\right)\,\mathrm{d}u$$
$$= \frac{1}{2}\int 1\,\mathrm{d}u - \int \frac{1}{u}\,\mathrm{d}u$$
$$= \frac{1}{2}u - \ln u = \frac{1}{2}(5+2t) - \ln(5+2t) + C$$

Aufgabe 15 (21) ($\star$)

$$\int \sqrt{\mathrm{e}^{3x} + \mathrm{e}^{2x}}\,\mathrm{d}x = \ldots$$

Substitution: $x(t) = \ln t \quad \Rightarrow \quad \mathrm{d}x = t^{-1}\,\mathrm{d}t$

$$\ldots = \int \sqrt{\mathrm{e}^{2x}\left(\mathrm{e}^{x}+1\right)}\,\mathrm{d}x$$
$$= \int \mathrm{e}^{x}\sqrt{\mathrm{e}^{x}+1}\,\mathrm{d}x = \int \mathrm{e}^{\ln t}\sqrt{\mathrm{e}^{\ln t}+1}t^{-1}\,\mathrm{d}t$$
$$= \int t\sqrt{t+1}t^{-1}\,\mathrm{d}t = \int (t+1)^{\frac{1}{2}}\,\mathrm{d}t$$
$$= \tfrac{2}{3}(t+1)^{\frac{3}{2}} + C_t = \tfrac{2}{3}\left(\mathrm{e}^{x}+1\right)^{\frac{3}{2}} + C$$

Aufgabe 16 (22) ($\star$)

$$\int \frac{1}{1+\sqrt{x}}\,\mathrm{d}x = \ldots$$

Substitution: $x(t) = t^2 \quad \Rightarrow \quad \mathrm{d}x = 2t\,\mathrm{d}t$

$$\ldots = \int \frac{1}{1+\sqrt{t^2}} \cdot 2t\,\mathrm{d}t = 2\int \frac{t}{1+t}\,\mathrm{d}t = \ldots$$

$u = 1+t \quad \Rightarrow \quad \mathrm{d}u = \mathrm{d}t$

$$\ldots = 2\int \frac{u-1}{u}\,\mathrm{d}u = 2\int 1\,\mathrm{d}u - 2\int \frac{1}{u}\,\mathrm{d}u = 2u - 2\ln|u| + C_u$$
$$= 2(1+t) - 2\ln|1+t| + C_t$$
$$= 2\big(1+\sqrt{x}\big) - 2\ln\big|1+\sqrt{x}\big| + C$$

Aufgabe 17 (24)

$$\int_0^\pi x\sin x\,\mathrm{d}x = \ldots$$

$$f'(x) = \sin x \quad\Rightarrow\quad f(x) = -\cos x$$
$$g(x) = x \quad\Rightarrow\quad g'(x) = 1$$

$$\ldots = \big[-x\cos x\big]_0^\pi + \int_0^\pi 1\cdot\cos x\mathrm{d}x = \big(-\pi\cos\pi - 0\big) + \big[\sin x\big]_0^\pi$$
$$= -\pi\cdot(-1) + \sin\pi - \sin 0 = \pi$$

Aufgabe 18 (25)

$$\int \mathrm{e}^x\sin x\,\mathrm{d}x = \ldots$$

$$f'(x) = \mathrm{e}^x \quad\Rightarrow\quad f(x) = \mathrm{e}^x$$
$$g(x) = \sin x \quad\Rightarrow\quad g'(x) = \cos x$$

$$\ldots = \mathrm{e}^x\sin x - \int \mathrm{e}^x\cos x\,\mathrm{d}x$$

$$f'(x) = \mathrm{e}^x \quad\Rightarrow\quad f(x) = \mathrm{e}^x$$
$$g(x) = \cos x \quad\Rightarrow\quad g'(x) = -\sin x$$

$$\ldots = \mathrm{e}^x\sin x - \left(\mathrm{e}^x\cos x - \int \mathrm{e}^x(-\sin x)\,\mathrm{d}x\right)$$
$$= \mathrm{e}^x\sin x - \mathrm{e}^x\cos x - \int \mathrm{e}^x\sin x\,\mathrm{d}x$$

Addiere auf beiden Seiten $\int \mathrm{e}^x\sin x\,\mathrm{d}x$:

$$2\int \mathrm{e}^x\sin x\,\mathrm{d}x = \mathrm{e}^x\sin x - \mathrm{e}^x\cos x$$
$$\int \mathrm{e}^x\sin x\,\mathrm{d}x = \frac{1}{2}\big(\mathrm{e}^x\sin x - \mathrm{e}^x\cos x\big) + C$$

Aufgabe 19 (26)

$$\int \cos^2 x\,\mathrm{d}x = \int \cos x\cos x\,\mathrm{d}x = \ldots$$

$f'(x) = \cos x \quad \Rightarrow \quad f(x) = \sin x$
$g(x) = \cos x \quad \Rightarrow \quad g'(x) = -\sin x$

$$\ldots = \sin x \cos x - \int \sin x(-\sin x)\,\mathrm{d}x$$
$$= \sin x \cos x + \int \sin^2 x\,\mathrm{d}x$$
$$= \sin x \cos x + \int (1 - \cos^2 x)\,\mathrm{d}x$$
$$= \sin x \cos x + \int 1\,\mathrm{d}x - \int \cos^2 x\,\mathrm{d}x$$

Addiere auf beiden Seiten $\int \cos^2 x\,\mathrm{d}x$:

$$2\int \cos^2 x\,\mathrm{d}x = \sin x \cos x + x$$
$$\int \cos^2 x\,\mathrm{d}x = \frac{1}{2}(\sin x \cos x + x) + C$$

Aufgabe 20 (27)

$$\int x^2 \mathrm{e}^{-x}\,\mathrm{d}x = \ldots$$

$f'(x) = \mathrm{e}^{-x} \quad \Rightarrow \quad f(x) = -\mathrm{e}^{-x}$
$g(x) = x^2 \quad \Rightarrow \quad g'(x) = 2x$

$$\ldots = -x^2\mathrm{e}^{-x} + 2\int x\mathrm{e}^{-x}\,\mathrm{d}x$$

$f'(x) = \mathrm{e}^{-x} \quad \Rightarrow \quad f(x) = -\mathrm{e}^{-x}$
$g(x) = x \quad \Rightarrow \quad g'(x) = 1$

$$\ldots = -x^2\mathrm{e}^{-x} + 2\left(-x\mathrm{e}^{-x} + \int 1\mathrm{e}^{-x}\,\mathrm{d}x\right)$$
$$= -x^2\mathrm{e}^{-x} - 2x\mathrm{e}^{-x} - \mathrm{e}^{-x} + C$$
$$= -(x^2 + 2x + 2)\mathrm{e}^{-x} + C$$

Aufgabe 21 (30)

$$\int_1^{\mathrm{e}} x \ln|x|\mathrm{d}x = \ldots$$

$f'(x) = x \quad \Rightarrow \quad f(x) = \frac{1}{2}x^2$
$g(x) = \ln|x| \quad \Rightarrow \quad g'(x) = x^{-1}$

$$\ldots = \left[\frac{1}{2}x^2 \ln|x|\right]_1^{\mathrm{e}} - \int_0^{\mathrm{e}} \frac{1}{2}x^2 \cdot \frac{1}{x}\,\mathrm{d}x = \frac{1}{2}\mathrm{e}^2 - \frac{1}{4}\left[x^2\right]_1^{\mathrm{e}} = \frac{1}{4}\mathrm{e}^2 + \frac{1}{4}$$

Aufgabe 22 (31)

$$\int \ln^2 x \,\mathrm{d}x = \int 1 \cdot \ln^2 x \,\mathrm{d}x = \ldots$$

$$\begin{aligned} f'(x) &= 1 & \Rightarrow \quad f(x) &= x \\ g(x) &= \ln^2 x & \Rightarrow \quad g'(x) &= 2\ln x \cdot x^{-1} \end{aligned}$$

$$\begin{aligned} \ldots &= x\ln^2 x - 2\int x \ln x \cdot x^{-1}\,\mathrm{d}x = x\ln^2 x - 2\int \ln x \,\mathrm{d}x \\ &= x\ln^2 x - 2x(\ln x - 1) + C = x(\ln^2 x - 2\ln x + 2) + C \end{aligned}$$

Aufgabe 23 (32)

$$\int \frac{\ln x}{x^2}\,\mathrm{d}x = \ldots$$

$$\begin{aligned} f'(x) &= x^{-2} & \Rightarrow \quad f(x) &= -x^{-1} \\ g(x) &= \ln x & \Rightarrow \quad g'(x) &= x^{-1} \end{aligned}$$

$$\begin{aligned} \ldots &= -x^{-1}\ln x + \int x^{-1} \cdot x^{-1}\,\mathrm{d}x \\ &= -x^{-1}\ln x + \int x^{-2}\,\mathrm{d}x \\ &= -x^{-1}\ln x - x^{-1} + C \end{aligned}$$

Aufgabe 24 (28)

$$\int x^3 \cdot \mathrm{e}^x \,\mathrm{d}x = \mathrm{e}^x(x^3 - 3x^2 + 6x - 6) + C$$

Aufgabe 25 (33)

$$\int x^4 \ln x \,\mathrm{d}x = \tfrac{1}{5}x^5\big(\ln x - \tfrac{1}{5}\big) + C$$

Aufgabe 26 (34)

$$\begin{aligned} \int \sin^3 x \,\mathrm{d}x &= -\tfrac{1}{3}\sin^2 x \cos x + \tfrac{2}{3}\int \sin x \,\mathrm{d}x \\ &= -\tfrac{1}{3}\sin^2 x \cos x - \tfrac{2}{3}\cos x + C \end{aligned}$$

Aufgabe 27 (35)

$$\int \cos^4 x \,\mathrm{d}x = \tfrac{1}{4}\sin x\cos^3 x + \tfrac{3}{4}\int \cos^2 x \,\mathrm{d}x$$

$$= \tfrac{1}{4}\sin x\cos^3 x + \tfrac{3}{4}\cdot\tfrac{1}{2}(x + \sin x\cos x) + C$$

$$= \tfrac{1}{4}\sin x\cos^3 x + \tfrac{3}{8}\sin x\cos x + \tfrac{3}{8}x + C$$

Aufgabe 28 (36)

$$\int (2x^2 + 4x - 5)\mathrm{e}^{2x}\,\mathrm{d}x$$

$$= \mathrm{e}^{2x}\left[\tfrac{1}{2}(2x^2 + 4x - 5) - \tfrac{1}{4}(4x + 4) + \tfrac{1}{8}\cdot 4\right] + C$$

$$= \mathrm{e}^{2x}\left[x^2 + 2x - \tfrac{5}{2} - x - 1 + \tfrac{1}{2}\right] + C$$

$$= \mathrm{e}^{2x}\left[x^2 + x - 3\right] + C$$

Aufgabe 29 (37)

$$\int_0^1 \frac{\mathrm{e}^x}{1 + \mathrm{e}^x}\,\mathrm{d}x$$

Substitution: $u = 1 + \mathrm{e}^x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{\mathrm{e}^x}\,\mathrm{d}u$

$$\ldots = \int_1^{1+\mathrm{e}} \frac{\mathrm{e}^x}{u}\cdot\frac{1}{\mathrm{e}^x}\,\mathrm{d}u = \int_1^{1+\mathrm{e}} \frac{1}{u}\,\mathrm{d}u = \big[\ln|u|\big]_1^{1+\mathrm{e}} = \ln(1+\mathrm{e}) - \ln 2 = \ln\frac{1+\mathrm{e}}{2}$$

Aufgabe 30 (38)

$$\int_0^1 (4x+1)^3\mathrm{d}x = \ldots$$

Substitution: $u = 4x + 1 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = 4 \quad \Rightarrow \quad \mathrm{d}x = \tfrac{1}{4}\mathrm{d}u$

$$\ldots = \int_{u(0)}^{u(1)} u^3\cdot\frac{1}{4}\mathrm{d}u = \frac{1}{4}\int_1^5 u^3\mathrm{d}u = \frac{1}{16}\left[u^4\right]_1^5 = \frac{1}{16}(625 - 1) = 39$$

Aufgabe 31 (39)

$$\int_0^1 x\mathrm{e}^{-x}\,\mathrm{d}x = \ldots$$

$f'(x) = \mathrm{e}^{-x} \quad \Rightarrow \quad f(x) = -\mathrm{e}^{-x}$

$g(x) = x \quad \Rightarrow \quad g'(x) = 1$

$$\ldots = \left[-x\mathrm{e}^{-x}\right]_0^1 - \int_0^1 \left(-\mathrm{e}^{-x}\right)\mathrm{d}x = -\mathrm{e}^{-1} + \int_0^1 \mathrm{e}^{-x}\,\mathrm{d}x = \mathrm{e}^{-1} - \left[\mathrm{e}^{-x}\right]_0^1 = 1 - 2\mathrm{e}^{-1}$$

Aufgabe 32 (40)

$$\int_0^1 \frac{3x}{x^2+9}\,\mathrm{d}x = \dots$$

Substitution: $u = x^2 + 9 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = 2x \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{2x}\mathrm{d}u$

$$\dots = \int_{u(0)}^{u(1)} \frac{3x}{u} \cdot \frac{1}{2x}\mathrm{d}u = \frac{3}{2}\int_9^{10} \frac{1}{u}\mathrm{d}u = \frac{3}{2} \cdot [\ln|u|]_9^{10} = \frac{3}{2}\big(\ln(10) - \ln(9)\big) = \frac{3}{2}\ln\left(\frac{10}{9}\right)$$

Aufgabe 33 (41)

$$\int_{-1}^1 x^2 \cdot \mathrm{e}^{(x^3)}\mathrm{d}x = \dots$$

Substitution: $u = x^3 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = 3x^2 \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{3x^2}\mathrm{d}u$

$$\dots = \int_{u(-1)}^{u(1)} x^2\mathrm{e}^u \cdot \frac{1}{3x^2}\mathrm{d}u = \frac{1}{3}\int_{-1}^1 \mathrm{e}^u\mathrm{d}u = \frac{1}{3}\,[\mathrm{e}^u]_{-1}^1 = \frac{1}{3}\big(\mathrm{e} - \mathrm{e}^{-1}\big)$$

Aufgabe 34 (42)

$$\int_0^{\pi/2} \sin^2 x \cdot \cos x\,\mathrm{d}x = \dots$$

$u = \sin x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \cos x \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{\cos x}\mathrm{d}u$

$$\dots = \int_0^1 u^2 \cdot \cos x \cdot \frac{1}{\cos x}\mathrm{d}u = \int_0^1 u^2\mathrm{d}u = \frac{1}{3}\left[u^3\right]_0^1 = \frac{1}{3}(1-0) = \frac{1}{3}$$

Aufgabe 35 (43)

$$\int_1^2 x^2 \ln x\,\mathrm{d}x = \dots$$

$f'(x) = x^2 \qquad \Rightarrow \quad f(x) = \frac{1}{3}x^3$

$g(x) = \ln(x) \quad \Rightarrow \quad g'(x) = \frac{1}{x}$

$$\dots = \left[\frac{1}{3}x^3 \ln x\right]_1^2 - \int_1^2 \frac{1}{3}x^3\frac{1}{x}\,\mathrm{d}x = \frac{8}{3}\ln 2 - \frac{1}{3}\int_1^2 x^2\,\mathrm{d}x = \frac{8}{3}\ln 2 - \frac{1}{9}\left[x^3\right]_1^2 = \frac{8}{3}\ln 2 - \frac{7}{9}$$

Aufgabe 36 (45)

$$\int_1^{\mathrm{e}} \frac{\sqrt{\ln x}}{x}\mathrm{d}x = \dots$$

Substitution: $u(x) = \ln(x) \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x} \quad \Rightarrow \quad \mathrm{d}x = x\,\mathrm{d}u$

$$\dots = \int_{u(1)}^{u(\mathrm{e})} \frac{\sqrt{u}}{x} \cdot x\,\mathrm{d}u = \int_0^1 \sqrt{u}\,\mathrm{d}u = \left[\frac{2}{3}\,x^{3/2}\right]_0^1 = \frac{2}{3}$$

Aufgabe 37 (46)

$$\int 1 \cdot \arccos x\,\mathrm{d}x = \dots$$

$$f'(x) = 1 \quad \Rightarrow \quad f(x) = x$$
$$g(x) = \arccos x \quad \Rightarrow \quad g'(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\dots = x \arccos x - \int x \cdot \frac{-1}{\sqrt{1-x^2}}\,\mathrm{d}x = \dots$$

$$u(x) = 1 - x^2 \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = -2x \quad \Rightarrow \quad \mathrm{d}x = -\tfrac{1}{2x}\,\mathrm{d}u$$

$$\begin{aligned}
\dots &= x \arccos x - \int x \cdot \frac{1}{u} \cdot \frac{-1}{2x}\,\mathrm{d}u \\
&= x \arccos x + \int \frac{1}{2\sqrt{u}}\,\mathrm{d}u \\
&= x \arccos x + \sqrt{u} + C = x \arccos x + \sqrt{1-x^2} + C
\end{aligned}$$