Aufgabe 1

(a) $(x - 6)(x - 8) = x^2 - 8x - 6x + 48 = x^2 - 14x + 48 = 0$

(b) $\left(x - \frac{1}{3}\right)\left(x - \frac{1}{3}\right) = x^2 - \frac{1}{3}x - \frac{1}{3}x + \frac{1}{9} = x^2 - \frac{2}{3}x + \frac{1}{9} = 0$

(c) $(x - 7)(x + 7) = x^2 - 49 = 0$

(d) $\left(x - \left(1 + \sqrt{3}\right)\right)\left(x - \left(1 - \sqrt{3}\right)\right)$

$\quad = \left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{3}\right)$

$\quad = \left((x - 1) - \sqrt{3}\right)\left((x - 1) + \sqrt{3}\right)$

$\quad = (x - 1)^2 - 3 = x^2 - 2x + 1 - 3 = x^2 - 2x - 2 = 0$

Aufgabe 2

(a) $a\left(x - \frac{5}{6}\right)\left(x - \frac{3}{4}\right) = a\left(x^2 - \frac{3}{4}x - \frac{5}{6}x + \frac{15}{24}\right) = a\left(x^2 - \frac{19}{12}x + \frac{5}{8}\right)$

Wähle $a = \text{kgV}(12, 8) = 24$:

$24x^2 - 38x + 15 = 0$

(b) $a\left(x + 2\right)\left(x - \frac{3}{4}\right) = a\left(x^2 - \frac{3}{4}x + 2x - \frac{3}{2}\right) = a\left(x^2 + \frac{5}{4}x - \frac{3}{2}\right)$

Wähle $a = \text{kgV}(4, 2) = 4$:

$4x^2 + 5x - 6 = 0$

Aufgabe 3

(a) $x^2 - x - 5700$

$D = b^2 - 4ac = 1 - 4 \cdot 1 \cdot 5700 = 1 + 22\,800 = 22\,801 = 151^2$

$x_1 = \dfrac{-b + \sqrt{D}}{2a} = \dfrac{1 + 151}{2} = 76$

$x_2 = \dfrac{-b - \sqrt{D}}{2a} = \dfrac{1 - 151}{2} = -75$

$x^2 - x - 5700 = (x - 76)(x + 75)$

(b) $x^2 + x - 1$

$$D = b^2 - 4ac = 1 - 4 \cdot 1 \cdot (-1) = 1 + 4 = 5$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-1 + \sqrt{5}}{2}$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-1 - \sqrt{5}}{2}$$

$$x^2 + x - 1 = \left(x - \frac{-1 + \sqrt{5}}{2} \right) \left(x - \frac{-1 - \sqrt{5}}{2} \right)$$

$$= \left(x + \frac{1}{2} - \frac{\sqrt{5}}{2} \right) \left(x + \frac{1}{2} + \frac{\sqrt{5}}{2} \right)$$

Aufgabe 4

(a) $6x^2 + x - 2 = 0$

$$D = b^2 - 4ac = 1 - 4 \cdot 6 \cdot (-2) = 49$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-1 + 7}{12} = \frac{6}{12} = \frac{1}{2}$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-1 - 7}{12} = \frac{-8}{12} = -\frac{2}{3}$$

$$6x^2 + x - 2 = 6 \left(x - \tfrac{1}{2} \right) \left(x + \tfrac{2}{3} \right)$$

(b) $-4x^2 + 11x + 45 = 0$

$$D = b^2 - 4ac = 121 - 4 \cdot (-4) \cdot 45 = 841 = 29^2$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{-11 + 29}{-8} = \frac{18}{-8} = -\frac{9}{4}$$

$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{-11 - 29}{-8} = \frac{-40}{-8} = 5$$

$$-4x^2 + 11x + 45 = -4 \left(x + \tfrac{9}{4} \right) \left(x - 5 \right)$$

Aufgabe 5

(a) $\dfrac{x^2 + 5x - 14}{x^2 - 7x + 10} = \dfrac{(x + 7)(x - 2)}{(x - 5)(x - 2)} = \dfrac{x + 7}{x - 5}$

(b) $\dfrac{6x^2 - 11x + 3}{8x^2 - 6x - 9}$

Zähler: $6x^2 - 11x + 3 = 0$; $D = 121 - 4 \cdot 6 \cdot 3 = 49$

$x_1 = \dfrac{11 + 7}{12} = \dfrac{3}{2}$; $x_2 = \dfrac{11 - 7}{12} = \dfrac{1}{3}$

Nenner: $8x^2 - 6x - 9 = 0$; $D = 36 - 4 \cdot 8 \cdot (-9) = 324 = 18^2$

$x_1 = \dfrac{6 + 18}{16} = \dfrac{3}{2}$; $x_2 = \dfrac{6 - 18}{16} = -\dfrac{3}{4}$

$\dfrac{6x^2 - 11x + 3}{8x^2 - 6x - 9} = \dfrac{6\left(x - \frac{3}{2}\right)\left(x - \frac{1}{3}\right)}{8\left(x - \frac{3}{2}\right)\left(x + \frac{3}{4}\right)} = \dfrac{3\left(x - \frac{1}{3}\right)}{4\left(x + \frac{3}{4}\right)} = \dfrac{3x - 1}{4x + 3}$