Integralrechnung (uneigentliche Integrale) Lösungen Übungen

Aufgabe 6.1

(a) $\lim\limits_{a\to\infty}\int_1^a \frac{1}{x^4}\mathrm{d}x = \lim\limits_{a\to\infty}\left[-\frac{1}{3x^3}\right]_1^a$

$= \lim\limits_{a\to\infty}\left(-\frac{1}{3a^3}+\frac{1}{3}\right) = \frac{1}{3}$

(b) $\lim\limits_{a\to\infty}\int_1^a \frac{1}{x^{1.1}}\mathrm{d}x = \lim\limits_{a\to\infty}\int_1^a x^{-1.1}\mathrm{d}x = \lim\limits_{a\to\infty}\left[-10x^{-0.1}\right]_1^a$

$= \lim\limits_{a\to\infty}\left(-\frac{10}{a^{0.1}}+\frac{10}{1^{0.1}}\right) = \lim\limits_{a\to\infty}\left(10-\frac{10}{a^{0.1}}\right) = 10$

(c) $\lim\limits_{a\to\infty}\int_3^a \frac{4+t}{t^3}\mathrm{d}t = \lim\limits_{a\to\infty}\int_3^a \left(\frac{4}{t^3}+\frac{1}{t^2}\right)\mathrm{d}t$

$= \lim\limits_{a\to\infty}\left[\frac{-2}{t^2}-\frac{1}{t}\right]_3^a = \lim\limits_{a\to\infty}\left[\left(-\frac{2}{a^2}-\frac{1}{a}\right)-\left(-\frac{2}{9}-\frac{1}{3}\right)\right]$

$= \lim\limits_{a\to\infty}\left[\frac{5}{9}-\frac{2}{a^2}-\frac{1}{a}\right] = \frac{5}{9}$

Aufgabe 6.2

(a) $\lim\limits_{a\to 0}\int_a^2 \frac{2}{x^2}\mathrm{d}x = \lim\limits_{a\to 0}\left[\frac{-2}{x}\right]_a^2 = \lim\limits_{a\to 0}\left(-1+\frac{2}{a}\right)$

existiert nicht

(b) $\lim\limits_{a\to 0}\int_a^4 \frac{2}{\sqrt{t}}\mathrm{d}t = \lim\limits_{a\to 0}\left[4\sqrt{t}\right]_a^4 = \lim\limits_{a\to 0}\left(8-4\sqrt{t}\right) = 8$

(c) $\lim\limits_{a\to 0}\int_a^4 u^{-\frac{3}{2}}\mathrm{d}u = \left[-2u^{-\frac{1}{2}}\right]_a^4 = \lim\limits_{a\to 0}\left(-1+\frac{2}{\sqrt{a}}\right)$

existiert nicht

(d) $\lim\limits_{a\to 0}\int_a^8 u^{-\frac{2}{3}}\mathrm{d}u = \lim\limits_{a\to 0}\left[3u^{\frac{1}{3}}\right]_a^8 = \lim\limits_{a\to 0}\left[6-3a^{\frac{1}{3}}\right] = 6$

Aufgabe 6.3

(a) $\lim\limits_{a\to\infty}\int_0^a \mathrm{e}^{-x}\mathrm{d}x = \lim\limits_{a\to\infty}\left[-\mathrm{e}^{-x}\right]_0^a = \lim\limits_{a\to\infty}(-\mathrm{e}^{-a}+\mathrm{e}^0)$

$= \lim\limits_{a\to\infty}(1-\mathrm{e}^{-a}) = \lim\limits_{a\to\infty}\big(1-\mathrm{e}^{-a}\big) = 1$

(b) $\lim\limits_{a\to-\infty}\int_a^0 \mathrm{e}^{-t}\mathrm{d}t = \lim\limits_{a\to-\infty}\left[-\mathrm{e}^{-t}\right]_a^0 = \lim\limits_{a\to-\infty}\big(-1+\mathrm{e}^{-a}\big)$

existiert nicht

(c) $\lim\limits_{a\to\infty}\int_0^a z\mathrm{e}^{-z}\mathrm{d}z = \lim\limits_{a\to\infty}\left[\mathrm{e}^{-z}(-z-1)\right]_0^a$

$\lim\limits_{a\to\infty}\left[\mathrm{e}^{-a}(-a-1)-\mathrm{e}^0(0-1)\right] = 1$

(d) $\lim\limits_{a\to\infty}\int_0^a y^2\mathrm{e}^{-y}\mathrm{d}y = \lim\limits_{a\to\infty}\left[\mathrm{e}^{-y}\big(-y^2-2y-1\big)\right]_0^a$

$= \lim\limits_{a\to\infty}\left[\mathrm{e}^{-a}(-a^2-2a-1)-(-1)\right] = 2$

Aufgabe 6.4

(a) $\lim\limits_{a\to-\infty}\int_a^{-2}\frac{1}{(y+1)^3}\mathrm{d}y = \lim\limits_{a\to-\infty}\left[\frac{-1}{2}\cdot(y+1)^{-2}\right]_a^{-2}$

$= -\frac{1}{2}\lim\limits_{a\to-\infty}\left[\frac{1}{(-2+1)^2}+\frac{1}{(a+1)^2}\right] = -\frac{1}{2}$

(b) $\lim\limits_{a\to-1}\int_a^3\frac{1}{z+1}\mathrm{d}z = \lim\limits_{a\to-1}\left[\ln(z+1)\right]_a^3$

$= \lim\limits_{a\to-1}\left[\ln 4-\ln(a+1)\right]$

existiert nicht

(c) $\lim\limits_{a\to\infty}2\int_{-a}^a\frac{1}{z^2+1}\mathrm{d}z = 2\lim\limits_{a\to\infty}\left[\arctan(z)\right]_{-a}^a$

$= 2\lim\limits_{a\to\infty}\left[\arctan(a)-\arctan(-a)\right] = 2\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = 2\pi$

Aufgabe 6.5

(a) $\lim\limits_{a\to\infty}\pi\int_1^a\left(\frac{1}{x^3}\right)^2\mathrm{d}x = \pi\lim\limits_{a\to\infty}\int_1^a\frac{1}{x^6}\mathrm{d}x$

$= \pi\lim\limits_{a\to\infty}\left[\frac{-1}{5\,x^5}\right]_1^a = \pi\lim\limits_{a\to\infty}\left(\frac{1}{5}-\frac{1}{5a^5}\right) = \frac{\pi}{5}$

(b) $\lim_{a\to\infty} \pi \int_1^a \left(\frac{\sqrt{x}}{x^2}\right)^2 \mathrm{d}x = \pi \lim_{a\to\infty} \int_1^a \frac{1}{x^3}\mathrm{d}x$

$$= \pi \lim_{a\to\infty} \left[\frac{-1}{2\,x^2}\right]_1^a = \pi \lim_{a\to\infty} \left(\frac{1}{2} - \frac{1}{2a^2}\right) = \frac{\pi}{2}$$