Aufgabe 1

(a) $\mathrm{d}x = A\omega \sin(\omega t - \delta)\,\mathrm{d}t$

(b) $\mathrm{d}W = m \cdot g \cdot \mathrm{d}h$

(c) $\mathrm{d}s = (a_0 t + v_0)\,\mathrm{d}t$

(d) $\mathrm{d}p = m \cdot \mathrm{d}v(t) = m \cdot \frac{\mathrm{d}v}{\mathrm{d}t} \cdot \mathrm{d}t = m \cdot a(t) \cdot \mathrm{d}t$

Aufgabe 2

(a) $\mathrm{d}s = (v_0 + at)\,\mathrm{d}t$

(b) $\mathrm{d}F = -G\,\frac{m_1 m_2}{2r^3}\,\mathrm{d}r$

(c) $\mathrm{d}p = -\frac{p_0 \alpha \beta}{T_0}\left(1 - \frac{\alpha \cdot h}{T_0}\right)^{\beta - 1}\,\mathrm{d}h$

Aufgabe 3

$s = (1.8 \pm 0.09)\,\mathrm{km}$

$t = (0.47 \pm 0.024)\,\mathrm{h}$

$v = \frac{s}{t}$

$$\Delta v \approx \left|\frac{\partial v}{\partial s}\right| \cdot \Delta s + \left|\frac{\partial v}{\partial t}\right| \cdot \Delta t = \left|\frac{1}{t}\right| \cdot \Delta s + \left|-\frac{s}{t^2}\right| \cdot \Delta t$$

$$= \frac{1}{0.47\,\mathrm{h}} \cdot 0.09\,\mathrm{km} + \frac{1.8\,\mathrm{km}}{0.47^2\,\mathrm{h}^2} \cdot 0.024\,\mathrm{h} = 0.39\,\tfrac{\mathrm{km}}{\mathrm{h}}$$

$$v = \frac{s}{t} = \frac{1.8\,\mathrm{km}}{0.47\,\mathrm{h}} = 3.83\,\tfrac{\mathrm{km}}{\mathrm{h}}$$

$v = (3.83 \pm 0.39)\,\tfrac{\mathrm{km}}{\mathrm{h}}$ (absolut)

$\frac{\Delta v}{v} = \frac{0.39}{3.83} = 0.10$ (relativ)

Aufgabe 4

- Bett: $s_1 = 2\,\mathrm{m} \pm 0.03\,\mathrm{m}$
- Schrank: $s_2 = 0.8\,\mathrm{m} \pm 0.03\,\mathrm{m}$
- Tisch: $s_3 = 1.0\,\mathrm{m} \pm 0.03\,\mathrm{m}$

$$s = s_1 + s_2 + s_3$$

$$\Delta s = \left|\frac{\partial s}{\partial s_1}\right| \cdot \Delta s_1 + \left|\frac{\partial s}{\partial s_2}\right| \cdot \Delta s_2 + \left|\frac{\partial s}{\partial s_3}\right| \cdot \Delta s_3$$

$$= 1 \cdot \Delta s_1 + 1 \cdot \Delta s_2 + 1 \cdot \Delta s_3 = 3 \cdot 0.03\,\mathrm{m} = 0.09\,\mathrm{m}$$

$$s = s_1 + s_2 + s_3 = 2.0 + 0.8 + 1.0 = 3.8\,\mathrm{m}$$

$$s = (3.80 \pm 0.09)\,\mathrm{m}$$

Bei einer Wandlänge von $3.85\,\mathrm{m}$ passen die drei Möbelstücke möglicherweise nicht mehr nebeneinander an die Wand.

Aufgabe 5

$$F(A, B) = c \cdot A \cdot B$$

$$\Delta F = \left|\frac{\partial F}{\partial A}\right| \cdot \Delta A + \left|\frac{\partial F}{\partial B}\right| \cdot \Delta B = |c \cdot B| \cdot \Delta A + |c \cdot A| \cdot \Delta B$$

$$\frac{\Delta F}{|F|} = \frac{|c \cdot B| \cdot \Delta A + |c \cdot A| \cdot \Delta B}{|c \cdot A \cdot B|} = \frac{\Delta A}{|A|} + \frac{\Delta B}{|B|}$$

Aufgabe 6

(a) $u = \ln x \Rightarrow \mathrm{d}u = \frac{1}{x} \cdot \mathrm{d}x \Rightarrow \mathrm{d}x = x \cdot \mathrm{d}u$

$$\int \frac{\ln x}{x}\,\mathrm{d}x = \int \frac{u}{x} \cdot x\,\mathrm{d}u = \int u\,\mathrm{d}u$$

$$= \frac{1}{2}u^2 + \tilde{C} = \frac{1}{2}(\ln x)^2 + C$$

(b) $u = z^2 + z \Rightarrow \mathrm{d}u = (2z+1) \cdot \mathrm{d}z \Rightarrow \mathrm{d}z = \frac{1}{2z+1} \cdot \mathrm{d}u$

$$\int_1^2 \frac{2z+1}{z^2+z}\,\mathrm{d}z = \int_2^6 \frac{2z+1}{u} \cdot \frac{1}{2z+1}\,\mathrm{d}u = \int_2^6 \frac{1}{u}\,\mathrm{d}u$$

$$= \big[\ln u\big]_2^6 = \ln 6 - \ln 2 = \ln 3$$

(c) $u = \sin x \Rightarrow \mathrm{d}u = \cos x \cdot \mathrm{d}x \Rightarrow \mathrm{d}x = \frac{1}{\cos x} \cdot \mathrm{d}u$

$$\int_0^{\pi/2} \sin x \cdot \cos x\,\mathrm{d}x = \int_0^1 u \cdot \cos x \cdot \frac{1}{\cos x}\,\mathrm{d}u = \int_0^1 u\,\mathrm{d}u$$

$$= \left[\frac{1}{2}u^2\right]_0^1 = 0.5 - 0 = 0.5$$

Aufgabe 7

(a) $x(t) = \ln t \quad \Rightarrow \quad \mathrm{d}x = \frac{1}{t}\,\mathrm{d}t$

$x_1 = 0 = \ln t \Rightarrow t_1 = 1$
$x_2 = 1 = \ln t \Rightarrow t_2 = \mathrm{e}$

$$\begin{aligned}
\int_0^1 \mathrm{e}^x \sqrt{\mathrm{e}^x + 1}\,\mathrm{d}x &= \int_1^{\mathrm{e}} \mathrm{e}^{\ln t} \sqrt{\mathrm{e}^{\ln t} + 1} \cdot \frac{1}{t}\,\mathrm{d}t \\
&= \int_1^{\mathrm{e}} t\sqrt{t+1} \cdot \frac{1}{t}\,\mathrm{d}t \\
&= \int_1^{\mathrm{e}} \sqrt{t+1}\,\mathrm{d}t = \left[\tfrac{2}{3}(t+1)^{3/2}\right]_1^{\mathrm{e}} \\
&= \tfrac{2}{3}\left[(\mathrm{e}+1)^{3/2} - 2^{3/2}\right] \approx 2.89
\end{aligned}$$

(b) $x(t) = \mathrm{e}^t$
$\mathrm{d}x = \mathrm{e}^t\,\mathrm{d}t$

$x_1 = 1 = \mathrm{e}^t \Rightarrow t_1 = 0$
$x_2 = 3 = \mathrm{e}^t \Rightarrow t_2 = \ln 3$

$$\begin{aligned}
\int_1^3 \frac{1}{x}\ \ln x^2\,\mathrm{d}x = \int_0^{\ln 3} \frac{1}{\mathrm{e}^t}\ \ln\left(\mathrm{e}^t\right)^2 \cdot \mathrm{e}^t\,\mathrm{d}t &= \int_0^{\ln 3} 2t\,\mathrm{d}t \\
= \left[t^2\right]_0^{\ln 3} = (\ln 3)^2 = 1.207
\end{aligned}$$

(c) $x(t) = t^2$
$\mathrm{d}x = 2t\,\mathrm{d}t$

$x_1 = 0 = t^2 \Rightarrow t_1 = 0$
$x_2 = 1 = t^2 \Rightarrow t_2 = 1$

$$\begin{aligned}
\int_0^1 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x &= \int_0^1 \frac{1}{1+\sqrt{t^2}} \cdot 2t\,\mathrm{d}x \\
&= \int_0^1 \frac{2t}{1+t}\,\mathrm{d}t = \int_0^1 \left(2 - \frac{2}{t+1}\right)\,\mathrm{d}t \\
&= 2\int_0^1 1\,\mathrm{d}t - 2\int_0^1 \frac{1}{t+1}\,\mathrm{d}t \\
&= 2\left[t\right]_0^1 - 2\left[\ln(t+1)\right]_0^1 = 2 - 2\ln 2 = 0.614
\end{aligned}$$

Aufgabe 8

(a) $s = \int_5^9 \sqrt{1 + \left[(4)'\right]^2} \mathrm{d}x = \int_5^9 \sqrt{1} \mathrm{d}x = \left[x\right]_5^9$

$= 9 - 5 = 4$

(b) $s = \int_1^3 \sqrt{1 + \left[(2x)'\right]^2}\, \mathrm{d}x = \int_1^3 \sqrt{1 + 4}\, \mathrm{d}x$

$= \int_1^3 \sqrt{5}\, \mathrm{d}x = \sqrt{5}\left[x\right]_1^3 = 2\sqrt{5}$

(c) $s = \int_0^9 \sqrt{1 + \left[\left(\frac{2}{3}(x-1)^{\frac{3}{2}}\right)'\right]^2}\, \mathrm{d}x$

$= \int_0^9 \sqrt{1 + \left[(x-1)^{\frac{1}{2}}\right]^2}\, \mathrm{d}x$

$= \int_0^9 \sqrt{x}\, \mathrm{d}x = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^9 = 18$

(d) $s = \int_0^2 \sqrt{1 + (2x)^2}\, \mathrm{d}x = \int_0^2 \sqrt{1 + 4x^2}\, \mathrm{d}x \overset{\text{TR}}{\approx} 4.647$

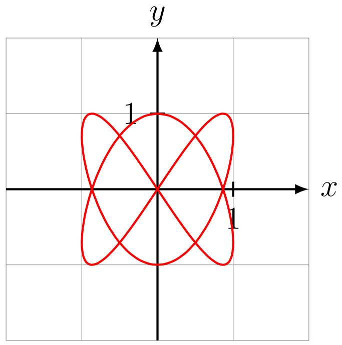
(e) $s = \int_0^4 \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2}\, \mathrm{d}x = \int_0^4 \sqrt{1 + \frac{1}{4x}}\, \mathrm{d}x \overset{\text{TR}}{\approx} 4.647$

(f) $s = \int_0^1 \sqrt{1 + (\mathrm{e}^x)^2}\, \mathrm{d}x = \int_0^1 \sqrt{1 + \mathrm{e}^{2x}}\, \mathrm{d}x \overset{\text{TR}}{\approx} 2.003$

Aufgabe 9

(a) $x(t) = \sin(2t) \quad \Rightarrow \quad \dot{x}(t) = 2\cos(2t)$

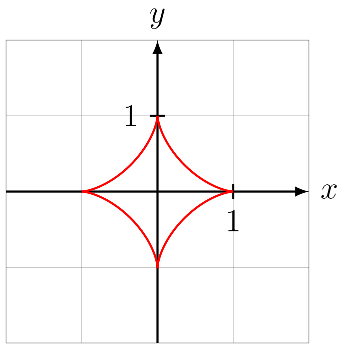
$y(t) = \sin(3t) \quad \Rightarrow \quad \dot{y}(t) = 3\cos(3t)$



$$s = \int_0^{2\pi} \sqrt{(2\cos 2t)^2 + (3\cos 3t)^2}\,\mathrm{d}t = \int_0^{2\pi} \sqrt{4\cos^2(2t) + 9\cos^2(3t)}\,\mathrm{d}t = 15.29$$

(b) $x(t) = \sin^3(t) \quad \Rightarrow \quad \dot{x}(t) = -3\sin^2(t)\cos(t)$

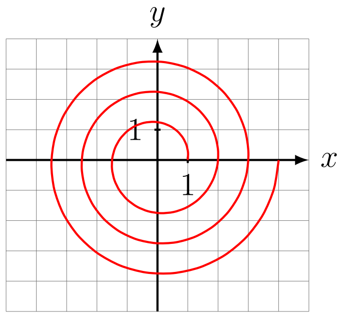
$y(t) = \cos^3(t) \quad \Rightarrow \quad \dot{y}(t) = 3\cos^2(t)\sin(t)$



$$s = \int_0^{2\pi} \sqrt{(2\cos 2t)^2 + (3\cos 3t)^2}\,\mathrm{d}t = \int_0^{2\pi} \sqrt{4\cos^2 2t + 9\cos^2 3t}\,\mathrm{d}t = 15.29$$

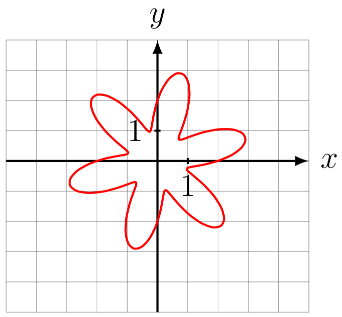
Aufgabe 10

(a) $r(\varphi) = 1 + \dfrac{\varphi}{2\pi} \quad \Rightarrow \quad r'(\varphi) = \dfrac{1}{2\pi}$



$$s = \int_{\varphi_1}^{\varphi_2} \sqrt{r(\varphi)^2 + r'(\varphi)^2} = \int_0^{6\pi} \sqrt{\left(1 + \frac{\varphi}{2\pi}\right)^2 + \frac{1}{4\pi^2}}\,\mathrm{d}t \approx 47.23$$

(b) $r(\varphi) = 2 + \sin(6\varphi) \quad \Rightarrow \quad r'(\varphi) = 6\cos(6\varphi)$



$$s = \int_{\varphi_1}^{\varphi_2} \sqrt{r(\varphi)^2 + r'(\varphi)^2}\,\mathrm{d}\varphi = \int_0^{2\pi} \sqrt{(2 + \sin 6\varphi)^2 + (6\cos 6\varphi)^2}\,\mathrm{d}\varphi \approx 28.18$$