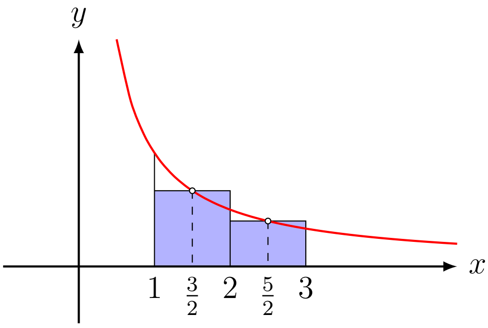
Aufgabe 1.1

(a) Graph:



$$I_0 = f(1.5)\cdot 1 + f(2.5)\cdot 1 = \frac{1}{3/2} + \frac{1}{5/2} = \frac{16}{15} = 1.0\overline{6}$$

(b) $I = \int_1^3 \frac{1}{x}\,\mathrm{d}x = \big[\ln(x)\big]_1^3 = \ln(3) - \ln(1) = \ln(3)$

rel. Fehler: $\dfrac{|I - I_0|}{I} = \left|\dfrac{\ln(3) - \frac{16}{15}}{\ln(3)}\right| = 0.029$

Aufgabe 1.2

[2nd] [$\int_{\square}^{\square} \square\, \mathrm{d}x$] $\displaystyle\int_0^{2\pi} \big(2 + \mathrm{e}^{-0.01x}\sin(5x)\big)\,\mathrm{d}x$ [enter] 12.579

Aufgabe 1.3

$$\int_1^5 f(x)\,\mathrm{d}x = 8,$$

$$\int_5^1 f(x)\,\mathrm{d}x = -8$$

Aufgabe 2.1

$$\int \frac{1}{x^2+4}\,\mathrm{d}x = \frac{1}{2}\arctan\frac{x}{2} + C \qquad \text{(S. 73)}$$

Aufgabe 2.2

$$\int_0^1 (x+1)^2\,\mathrm{d}x = \int_0^1 \big(x^2 + 2x + 1\big)\,\mathrm{d}x = \big[\tfrac{1}{3}x^3 + x^2 + x\big]_0^1 = \tfrac{7}{3}$$

Aufgabe 2.3

$$\int \frac{x^2 + x^4}{x^4}\,\mathrm{d}x = \int \left(\frac{1}{x^2} + 1\right)\mathrm{d}x = \int \big(x^{-2} + 1\big)\,\mathrm{d}x = -x^{-1} + x + C$$

Aufgabe 2.4

$f'(x) = 3x^2 + 2x + 1$ und $f(1) = 2$

$f(x) = x^3 + x^2 + x + C$

$f(1) = 1 + 1 + 1 + C = 2 \quad \Rightarrow \quad C = -1$

$f(x) = x^3 + x^2 + x - 1$

Aufgabe 3.1

$$\int_1^2 4x \cdot \mathrm{e}^{x^2} \, \mathrm{d}x = \ldots$$

Substitution: $u = x^2 \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2x \quad \Rightarrow \quad \mathrm{d}x = \dfrac{1}{2x} \, \mathrm{d}u$

$$\ldots = \int_{u(1)}^{u(4)} 4x \cdot \mathrm{e}^u \, \frac{1}{2x} \, \mathrm{d}u = 2 \int_1^4 \mathrm{e}^u \, \mathrm{d}u = 2 \big[\mathrm{e}^u\big]_1^4 = 2(\mathrm{e}^4 - \mathrm{e})$$

Aufgabe 3.2

$$\int x \cdot \cos(x) \, \mathrm{d}x = \ldots$$

partielle Integration: $f'(x) = \cos(x) \quad \Rightarrow \quad f(x) = \sin(x)$

$g(x) = x \quad \Rightarrow \quad g'(x) = 1$

$$\cdots = x \cdot \sin(x) + \int 1 \cdot \sin(x) \, \mathrm{d}x = x \sin(x) - \cos(x) + C$$

Aufgabe 3.3

$$\int \ln(x) \cdot 1 \, \mathrm{d}x = \ldots$$

partielle Integration: $f'(x) = 1 \quad \Rightarrow \quad f(x) = x$

$g(x) = \ln(x) \quad \Rightarrow \quad g'(x) = 1/x$

$$\ldots = x \ln(x) + \int \frac{1}{x} \cdot x \, \mathrm{d}x = x \ln(x) + \int 1 \, \mathrm{d}x = x \ln(x) + x + C$$

Aufgabe 3.4

$$\int_0^1 (2x + 3)^7 \, \mathrm{d}x = \ldots$$

Substitution: $u = 2x + 3 \quad \Rightarrow \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2 \quad \Rightarrow \quad \mathrm{d}x = \dfrac{1}{2} \, \mathrm{d}u$

$$\ldots = \int_{u(0)}^{u(1)} u^7 \cdot \tfrac{1}{2} \, \mathrm{d}u = \tfrac{1}{2} \int_3^5 u^7 \, \mathrm{d}u = \tfrac{1}{2} \big[\tfrac{1}{8} u^8\big]_3^5 = \tfrac{1}{16} \big[u^8\big]_3^5 = 24\,004$$

Aufgabe 3.5

$$\begin{aligned}\int \mathrm{e}^{2x}\big(x^2+x+1\big)\,\mathrm{d}x &= \mathrm{e}^{2x}\big(\tfrac{1}{2}(x^2+x+1)-\tfrac{1}{4}(2x+1)+\tfrac{1}{8}\cdot 2\big)\\ &= \mathrm{e}^{2x}\big(\tfrac{1}{2}x^2+\tfrac{1}{2}x+\tfrac{1}{2}-\tfrac{1}{2}x-\tfrac{1}{4}+\tfrac{1}{4}\big)=\tfrac{1}{2}\mathrm{e}^{2x}(x^2+1)+C\end{aligned}$$

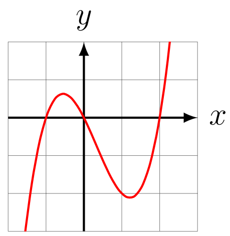
Aufgabe 4.1

$f\colon\, y = x^3 - x^2 - 2x = x(x-2)(x+1)$

(a) Nullstellen: $x_1 = -1$, $x2 = 0$, $x_3 = 2$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^3 = -\infty$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^3 = \infty$$



(b) Stammfunktion: $F(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$ (einmal notieren genügt)

$$A_1 = \int_{-1}^{0} f(x)\,\mathrm{d}x = \tfrac{5}{12}$$

$$A_2 = -\int_{0}^{2} f(x)\,\mathrm{d}x = \int_{2}^{0} f(x)\,\mathrm{d}x = \tfrac{8}{3}$$

$$A = A_1 + A_2 = \tfrac{37}{12}$$

Aufgabe 4.2

$$\begin{aligned}\int_{-1}^{2}\big(3x^2+p^2\big)\,\mathrm{d}x &= 21\\ \big[\tfrac{1}{9}x^3+p^2x\big]_{-1}^{2} &= 21\\ (8+2p^2)-(-1-p^2) &= 21\\ 3p^2+9 &= 21\\ 3p^2 &= 12\\ p &= \pm 2\end{aligned}$$

Aufgabe 4.3

$$\int_0^a \left(3x^2 - 6x + 4\right) \mathrm{d}x = 32$$

$$\left[x^3 - 3x^2 + 4x\right]_0^a = 32$$

$$a^3 - 3a^2 + 4a - 32 = 0$$

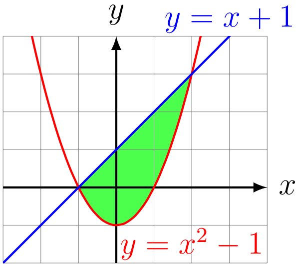
$$a = 4$$

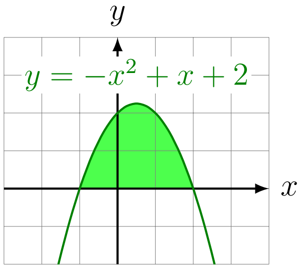
Aufgabe 4.4

Schnittstellen: $x^2 - 1 = x + 1$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0 \quad \Rightarrow \quad x_1 = -1,\ x_2 = 2$$





$$I = \int_{-1}^{2} \left((x+1) - (x^2 - 1)\right) \mathrm{d}x = \int_{-1}^{2} \left(-x^2 + x + 2\right) \mathrm{d}x$$

$$= \left[-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 2x\right]_{-1}^{2} = \tfrac{9}{2}$$

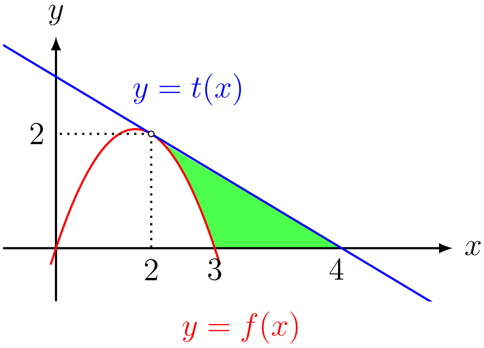
Aufgabe 4.5

$$f(x) = 3x - x^2 \quad \Rightarrow \quad f'(x) = 3 - 2$$

Die Tangente entspricht dem Taylorpolynom 1. Ordnung:

$$t(x) = f(2) + f'(2)(x-2) = 2 + (-1)(x-2) = 4 - x$$

Nullstelle von t: $x = 4$; linke Nullstelle von f: $x = 3$



$$A = A_{\text{Dreieck}} - A_{\text{Parabelstück}} = \tfrac{1}{2} \cdot 2 \cdot 2 - \int_2^3 (3x - x^2)\,\mathrm{d}x$$

$$= 2 - \left[\tfrac{3}{2}x^2 - \tfrac{1}{3}x^3\right]_2^3 = 2 - \tfrac{7}{6} = \tfrac{5}{6}$$