

Aufgabe 1

$$f(x) = x^3 - 2x + 4 \Rightarrow f'(x) = 3x^2 - 2$$

$$f(1) = 3, f'(1) = 1$$

$$T_1f(x; 1) = f(1) + f'(1)(x - 1) = 3 + 1 \cdot (x - 1) = x + 2$$

$$t: y = x + 2$$

Aufgabe 2

$$f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$$

$$f(4) = 2, f'(4) = \frac{1}{4}$$

$$T_1f(x; 4) = f(4) + f'(4)(x - 4) = 2 + \frac{1}{4}(x - 4) = \frac{1}{4}x + 1$$

$$t: y = \frac{1}{4}x + 1$$

Aufgabe 3

$$f(x) = \frac{2}{x} = 2x^{-1} \Rightarrow f'(x) = -2x^{-2} = \frac{-2}{x^2}$$

$$f(-2) = -1, f'(-2) = -\frac{1}{2}$$

$$T_1f(x; -2) = f(-2) + f'(-2)(x + 2) = -1 - \frac{1}{2}(x + 2) = -\frac{1}{2}x - 2$$

$$t: y = -\frac{1}{2}x - 2$$

Aufgabe 4

$$f(x) = (x - 1)e^x \Rightarrow f'(x) = 1 \cdot e^x + (x - 1)e^x = -xe^x$$

$$f(0) = (0 - 1)e^0 = -1, f'(0) = 0 \cdot e^0 = 0$$

$$T_1f(x; 0) = f(0) + f'(0)(x - 0) = -1$$

$$t: y = -1$$

Aufgabe 5

$$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$$

$$f(e) = \ln(e) = 1, f'(e) = \frac{1}{e}$$

$$T_1f(x; e) = f(e) + f'(e)(x - e) = 1 + \frac{1}{e}(x - e) = \frac{1}{e}x$$

$$t: y = \frac{1}{e}x$$

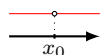
Aufgabe 6

$$f(x) = \cos(x) \Rightarrow f'(x) = -\sin(x) \Rightarrow f''(x) = -\cos(x)$$

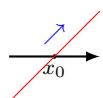
$$f(0) = \cos(0) = 1, f'(0) = -\sin(0) = 0, f''(0) = -\cos(0) = -1$$

$$T_2 f(x; 0) = f(0) + f'(0) \cdot x + \frac{1}{2} f''(0) x^2 = 1 - \frac{1}{2} x^2$$

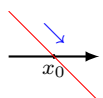
Aufgabe 7



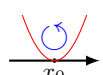
$$f(x_0) = y_0$$



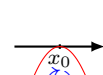
$$f'(x_0) > 0$$



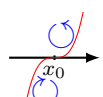
$$f'(x_0) < 0$$




$$f''(x_0) > 0$$



$$f''(x_0) < 0$$



$$f'''(x_0) > 0$$



$$f'''(x_0) < 0$$

vrnl: Krümmung wechselt von rechts nach links

vlnr: Krümmung wechselt von links nach rechts

(a) $T_3 f(x, 1) = 3 - 4(x - 1) + \frac{1}{2}(x - 1)^2 + \frac{1}{12}(x - 1)^3$

$P(1, 3)$: fallend ($f'(1) = -4$) und linksgekrümmt ($f''(1) = \frac{1}{2}$)

(b) $T_3 f(x, -2) = 5 - 3(x + 2)^2 + \frac{1}{4}(x + 2)^3$

$P(-2, 5)$: Hochpunkt ($f'(-2) = 0$, $f''(-2) = -3$)

(c) $T_3 f(x, 3) = -7 - \frac{1}{2}(x - 3)^2 - \frac{1}{8}(x - 3)^3$

$P(3, -7)$: Wendepunkt fallend *vlnr* ($f'(3) = -\frac{1}{2}$, $f''(3) = 0$, $f'''(3) = -3$)

(d) $T_3 f(x, -1) = \frac{5}{4} + \frac{1}{2}(x + 1)^2 + \frac{1}{16}(x + 1)^3$

$P(-1, \frac{5}{4})$: Tiefpunkt ($f'(-1) = 0$, $f''(-1) = \frac{1}{2}$)

(e) $T_3 f(x, 4) = 2 + \frac{1}{8}(x - 4)^3$

$P(4, 2)$: Terrassenpunkt *vrnl* ($f'(4) = f''(4) = 0$, $f'''(4) = \frac{1}{8}$)