Differenzialrechnung (Tangente und Normale) **Lösungen+** **Übungen**

Aufgabe 1

$f(x) = \sqrt{3\,x - 5}$

$x_0 = 2 \quad \Rightarrow \quad y_0 = f(2) = 1$

$m_t = f'(2) = \frac{3}{2}$ (Taschenrechner)

$y_0 = m_t \cdot x_0 + q_t$
$1 = \frac{3}{2} \cdot 2 + q_t$
$q_t = -2$

$\boxed{t\colon y = \frac{3}{2}x - 2}$

$m_t \cdot m_n = -1$
$\frac{3}{2} \cdot m_n = -1$
$m_n = -\frac{2}{3}$

$y_0 = m_n \cdot x_0 + q_n$
$1 = -\frac{2}{3} \cdot 2 + q_n$
$q_n = \frac{7}{3}$

$\boxed{n\colon y = -\frac{2}{3}x + \frac{7}{3}}$

Aufgabe 2

$f(x) = x^3 - x + 1$

$x_0 = -1 \quad \Rightarrow \quad y_0 = f(-1) = 1$

$m_t = f'(-1) = 2$ (Taschenrechner)

$y_0 = m_t \cdot x_0 + q_t$
$1 = 2 \cdot (-1) + q_t$
$q_t = 3$

$\boxed{t\colon y = 2x + 3}$

$m_t \cdot m_n = -1$
$2 \cdot m_n = -1$
$m_n = -\frac{1}{2}$

$y_0 = m_n \cdot x_0 + q_n$
$1 = -\frac{1}{2} \cdot (-1) + q_n$
$q_n = \frac{1}{2}$

$\boxed{n\colon y = -\frac{1}{2}x + \frac{1}{2}}$

Aufgabe 3

$$f(x) = \frac{5-x}{x-3}$$

$x_0 = 5 \quad \Rightarrow \quad y_0 = f(5) = 0$

$m_t = f'(5) = -\frac{1}{2}$ (Taschenrechner)

$$\begin{aligned} y_0 &= m_t \cdot x_0 + q_t \\ 0 &= -\tfrac{1}{2} \cdot 5 + q_t \\ q_t &= \tfrac{5}{2} \end{aligned}$$

$\boxed{t\colon y = -\frac{1}{2}x + \frac{5}{2}}$

$$\begin{aligned} m_t \cdot m_n &= -1 \\ -\tfrac{1}{2} \cdot m_n &= -1 \\ m_n &= 2 \end{aligned}$$

$$\begin{aligned} y_0 &= m_n \cdot x_0 + q_n \\ 0 &= 2 \cdot 5 + q_n \\ q_n &= -10 \end{aligned}$$

$\boxed{n\colon y = 2x - 10}$