

Aufgabe 1

(a) $x^2 + y^2 + z^2 - 4x + 2y - 10z + 26 = 0$

$$(x - 2)^2 + (y + 1)^2 + (z - 5)^2 = -26 + 4 + 1 + 25 = 4$$

$$M(2, -1, 5); r = 2$$

(b) $x^2 + y^2 + z^2 + 12x - 6z + 9 = 0$

$$(x + 6)^2 + y^2 + (z - 3)^2 = -9 + 36 + 9 = 36$$

$$M(-6, 0, 3), r = 6$$

(c) $x^2 + y^2 + z^2 - 14x + 4y + 53 = 0$

$$(x - 7)^2 + (y + 2)^2 + z^2 = -53 + 49 + 4 = 0$$

$$M(7, -2, 0), r = 0$$

(d) $2x^2 + 2y^2 + 2z^2 - 2x + 6y - 4z - 11 = 0$

$$x^2 + y^2 + z^2 - x + 3y - 2z = \frac{11}{2}$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 + (z - 1)^2 = \frac{1}{4} + \frac{9}{4} + 1 + \frac{22}{4} = 9$$

$$M\left(\frac{1}{2}, -\frac{3}{2}, 1\right), r = 3$$

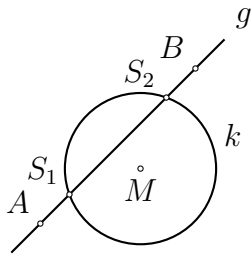
(e) $36x^2 + 36y^2 + 36z^2 + 48x - 108y + 60z - 103 = 0$

$$x^2 + y^2 + z^2 + \frac{4}{3}x - 3y + \frac{5}{3}z = \frac{103}{36}$$

$$\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{3}{2}\right)^2 + \left(z + \frac{5}{6}\right)^2 = \frac{103}{36} + \frac{4}{9} + \frac{9}{4} + \frac{25}{36} = \frac{25}{4}$$

$$M\left(-\frac{2}{3}, \frac{3}{2}, -\frac{5}{6}\right), r = \frac{5}{2}$$

Aufgabe 2



$$(a) \quad g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \Rightarrow \begin{aligned} x &= t + 5 \\ y &= -3t + 2 \\ z &= t + 1 \end{aligned}$$

$g \cap k$:

$$\begin{aligned} x^2 + y^2 + z^2 &= 41 \\ (t + 5)^2 + (-3t + 2)^2 + (t + 1)^2 &= 41 \\ 11t^2 + 30 &= 41 \\ 11t^2 &= 11 \\ t^2 &= 1 \\ t_1 = 1 &\Rightarrow S_1(6, -1, 2) \\ t_2 = -1 &\Rightarrow S_2(4, 5, 0) \end{aligned}$$

$$(b) \quad g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \Rightarrow \begin{aligned} x &= t + 5 \\ y &= 3t + 2 \\ z &= -t \end{aligned}$$

$g \cap k$:

$$\begin{aligned} x^2 + y^2 + z^2 &= 29 \\ (t + 5)^2 + (3t + 2)^2 + (-t)^2 &= 29 \\ 11t^2 + 22t + 29 &= 29 \\ 11t^2 + 22t &= 0 \\ 11t(t + 2) &= 0 \\ t_1 = 0 &\Rightarrow S_1(5, 2, 0) \\ t_2 = -2 &\Rightarrow S_2(3, -4, 2) \end{aligned}$$

$$(c) \quad g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 16 \end{pmatrix} + t \begin{pmatrix} 5 \\ 2 \\ 5 \end{pmatrix} \Rightarrow \begin{aligned} x &= 5t + 9 \\ y &= 2t + 5 \\ z &= 5t + 16 \end{aligned}$$

$g \cap k$:

$$\begin{aligned}
(x-2)^2 + (y+5)^2 + z^2 &= 81 \\
(5t+7)^2 + (2t+10)^2 + (5t+16)^2 &= 81 \\
54t^2 + 270t + 324 &= 0 \\
t^2 + 5t + 6 &= 0 \\
(t+2)(t+3) &= 0 \\
t_1 = -2 &\Rightarrow S_1(-1, 1, 6) \\
t_2 = -3 &\Rightarrow S_2(-6, -1, 1)
\end{aligned}$$

$$\text{(d) } g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \Rightarrow \begin{aligned} x &= 2t + 6 \\ y &= -t \\ z &= 0 \end{aligned}$$

$g \cap k$:

$$\begin{aligned}
(x+1)^2 + (y+4)^2 + (z-2)^2 &= 49 \\
(2t+7)^2 + (-t+4)^2 + (-2)^2 &= 49 \\
5t^2 + 20t + 20 &= 0 \\
t^2 + 4t + 4 &= 0 \\
(t+2)^2 &= 0 \\
t_{1,2} = 0 &\Rightarrow S_{1,2}(2, 2, 0)
\end{aligned}$$

Aufgabe 3

(a) $M(3, -1, 2)$

$$r = \frac{1}{2} |\vec{r}_B - \vec{r}_A| = \frac{1}{2} \sqrt{16 + 16 + 16} = \sqrt{12}$$

$$k: (x-3)^2 + (y+1)^2 + (z-2)^2 = 12$$

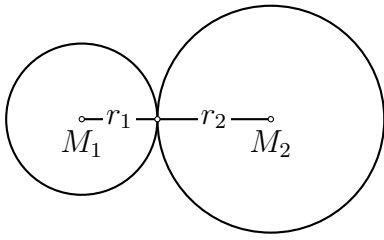
(b) $M(2, 3, 1)$

$$r = \frac{1}{2} |\vec{r}_B - \vec{r}_A| = \frac{1}{2} \sqrt{4 + 36 + 36} = \sqrt{19}$$

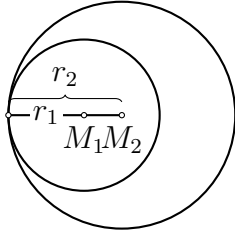
$$k: (x-2)^2 + (y-3)^2 + (z-1)^2 = 19$$

Aufgabe 4

Berührung aussen: $|M_1M_2| = r_1 + r_2$



Berührung innen: $|M_1M_2| = \max\{r_1, r_2\} - \min\{r_1, r_2\}$



$$k_1: (x - 1)^2 + (y + 3)^2 + z^2 = 54$$

$$k_2: (x - 11)^2 + (y - 2)^2 + (z + 5)^2 = -126 + 121 + 4 + 25 = 24$$

$$M_1(1, -3, 0), r_1 = \sqrt{54} = 3\sqrt{6}$$

$$M_2(11, 2, -5), r_2 = \sqrt{24} = 2\sqrt{6}$$

$$\overrightarrow{M_1M_2} = \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix} \Rightarrow |M_1M_2| = \sqrt{150} = \sqrt{6 \cdot 25} = 5\sqrt{6}$$

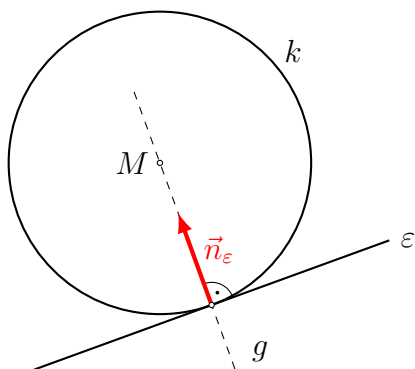
A horizontal line segment is shown with endpoints M_1 and M_2 . A point B is marked on the segment. The distance from M_1 to B is labeled $3\sqrt{6}$, and the distance from B to M_2 is labeled $2\sqrt{6}$.

(a) $|M_1M_2| = 5\sqrt{6} = 3\sqrt{6} + 2\sqrt{6} = r_1 + r_2$ (stimmt)

(b) $\vec{r}_B = \vec{r}_{M_1} + \frac{3}{5}\overrightarrow{M_1M_2} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} + \frac{3}{5} \begin{pmatrix} 10 \\ 5 \\ -5 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ -3 \end{pmatrix}$

$$\Rightarrow B(7, 0, -3)$$

Aufgabe 5



(a) $M(6, 5, -3)$; $\varepsilon: x - 2y + 2z + 4 = 0$

Die etwas aufwändigere Lösung:

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \\ -3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \Rightarrow \begin{array}{l} x = t + 6 \\ y = -2t + 5 \\ z = 2t - 3 \end{array}$$

$g \cap \varepsilon$:

$$\begin{aligned} x - 2y + 2z + 4 &= 0 \\ (t + 6) - 2(-2t + 5) + 2(2t - 3) + 4 &= 0 \\ 9t - 6 &= 0 \\ t &= \frac{2}{3} \end{aligned}$$

$$\overline{MB} = \frac{2}{3} \left| \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \right| = \frac{2}{3} \cdot 3 = 2$$

$$\Rightarrow (x - 6)^2 + (y - 5)^2 + (z + 3)^2 = 4$$

(b) $M(-4, 30, -5)$, $\varepsilon: 5x - 14y + 2z = 0$

Schnellere Lösung mit der Punkt-Ebene-Abstandsformel:

$$d(M, \varepsilon) = \frac{|5 \cdot (-4) - 14 \cdot 30 + 2 \cdot (-5) + 0|}{\sqrt{5^2 + (-14)^2 + 2^2}} = \frac{450}{15} = 30$$

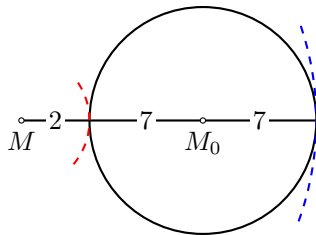
$$\Rightarrow (x + 4)^2 + (y - 30)^2 + (z + 5)^2 = 900$$

Aufgabe 6

(a) $k_0: (x - 2)^2 + (y + 3)^2 + (z - 1)^2 = 49$; $M_0(2, -3, 1)$, $r_0 = 7$

$$M(9, 1, 5) \quad \overrightarrow{M_0M} = \begin{pmatrix} 9 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 4 \end{pmatrix}$$

$$|M_0M| = \sqrt{49 + 16 + 16} = \sqrt{81} = 9$$



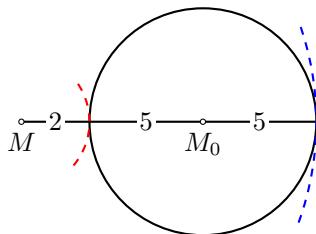
$$k_1: (x - 9)^2 + (y - 1)^2 + (z - 5)^2 = 4$$

$$k_2: (x - 9)^2 + (y - 1)^2 + (z - 5)^2 = 256$$

(b) $k_0: (x + 1)^2 + (y - 2)^2 + (z - 3)^2 = 25$; $M_0(-1, 2, 3)$, $r_0 = 5$

$$M(5, 4, 6) \quad \overrightarrow{M_0M} = \begin{pmatrix} 5 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$$

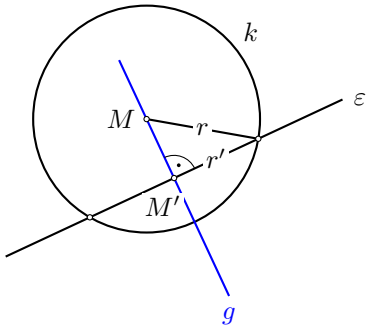
$$|M_0M| = \sqrt{36 + 4 + 9} = \sqrt{49} = 7$$



$$k_1: (x - 5)^2 + (y - 4)^2 + (z - 6)^2 = 4$$

$$k_2: (x - 5)^2 + (y - 4)^2 + (z - 6)^2 = 144$$

Aufgabe 7



$$x^2 + y^2 + z^2 - 2y - 22z - 103 = 0$$

$$x^2 + (y - 1)^2 + (z - 11)^2 = 103 + 1 + 121 = 225$$

$$\Rightarrow M(0, 1, 11), r = 15$$

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 11 \end{pmatrix} + t \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$

$g \cap \varepsilon$:

$$2(2t) + 2(2t + 1) - (-t + 11) - 18 = 0$$

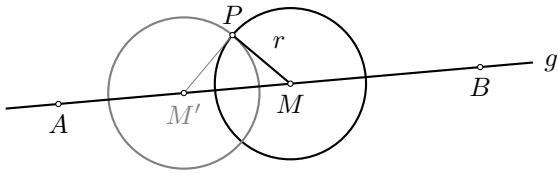
$$9t - 27 = 0$$

$$t = 3 \Rightarrow M'(6, 7, 8)$$

$$|MM'| = \left| \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 11 \end{pmatrix} \right| = \left| \begin{pmatrix} 6 \\ 6 \\ -3 \end{pmatrix} \right| = \sqrt{36 + 36 + 9} = 9$$

$$r' = \sqrt{r^2 - |MM'|^2} = \sqrt{15^2 - 9^2} = 12$$

Aufgabe 8



$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 7 \\ -2 \end{pmatrix} + t \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \begin{aligned} x &= -t \\ y &= 2t + 7 \\ z &= t - 2 \end{aligned}$$

$$M(-t, 2t + 7, t - 2) \in g$$

$$|\overrightarrow{PM}| = 3$$

$$(-t - 3)^2 + (2t + t)^2 + (t)^2 = 9$$

$$6t^2 + 30t + 36 = 0$$

$$t^2 + 5t + 6 = 0$$

$$(t + 2)(t + 3) = 0$$

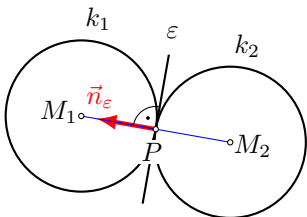
$$t_1 = -2 \Rightarrow M_1(2, 3, -4)$$

$$t_2 = -3 \Rightarrow M_1(3, 1, -5)$$

$$k_1: (x - 2)^2 + (y - 3)^2 + (z + 4)^2 = 9$$

$$k_2: (x - 3)^2 + (y - 1)^2 + (z + 5)^2 = 9$$

Aufgabe 9



(a) $r = 18$, $A(0, -2, 4)$, $B(-1, 2, -2)$, $C(-10, 6, 0)$, $P(3, -2, z)$

$$\vec{AB} = \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix}, \vec{AC} = \begin{pmatrix} -10 \\ 8 \\ -4 \end{pmatrix} \Rightarrow \vec{AB} \times \vec{AC} = \begin{pmatrix} 32 \\ 56 \\ 32 \end{pmatrix} = 8 \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix}$$

$$\varepsilon: 4x + 7y + 4z - 2 = 0$$

$$P \in \varepsilon: 4 \cdot 3 + 7 \cdot (-2) + 4z - 2 = 0$$

$$4z - 4 = 0$$

$$z = 1 \Rightarrow P(2, -2, 1)$$

$$\vec{r}_P + \frac{r}{|\vec{n}|} \vec{n} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} + \frac{18}{9} \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 11 \\ 12 \\ 9 \end{pmatrix} \Rightarrow M_1(11, 12, 9)$$

$$k_1: (x - 11)^2 + (y - 12)^2 + (z - 9)^2 = 324$$

$$\vec{r}_P - \frac{r}{|\vec{n}|} \vec{n} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} - \frac{18}{9} \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -16 \\ -7 \end{pmatrix} \Rightarrow M_2(-5, -16, -7)$$

$$k_2: (x + 5)^2 + (y + 16)^2 + (z + 7)^2 = 324$$

(b) $r = 42$; $A(6, 1, 0)$, $B(0, -3, 7)$, $C(-3, -3, 1)$; $P(10, y, 8)$

$$\vec{AB} = \begin{pmatrix} -6 \\ -4 \\ 7 \end{pmatrix}, \vec{AC} = \begin{pmatrix} -9 \\ -4 \\ 1 \end{pmatrix} \Rightarrow \vec{AB} \times \vec{AC} = \begin{pmatrix} 24 \\ -57 \\ -12 \end{pmatrix} = 3 \begin{pmatrix} 8 \\ -19 \\ -4 \end{pmatrix}$$

$$d = -\vec{n} \cdot \vec{r}_A = -29 \Rightarrow \varepsilon: 8x - 19y - 4z - 29 = 0$$

$$P \in \varepsilon: 8 \cdot 10 - 19 \cdot y - 4 \cdot 8 - 29 = 0$$

$$19y - 19 = 0$$

$$z = 1 \Rightarrow P(10, 1, 8)$$

$$\vec{r}_P + \frac{r}{|\vec{n}|} \vec{n} = \begin{pmatrix} 10 \\ 1 \\ 8 \end{pmatrix} + \frac{42}{21} \begin{pmatrix} 8 \\ -19 \\ -4 \end{pmatrix} = \begin{pmatrix} 26 \\ -37 \\ 0 \end{pmatrix} \Rightarrow M_1(26, -37, 0)$$

$$k_1: (x - 26)^2 + (y - 37)^2 + z^2 = 1764$$

$$\vec{r}_P - \frac{r}{|\vec{n}|} \vec{n} = \begin{pmatrix} 10 \\ 1 \\ 8 \end{pmatrix} - \frac{42}{21} \begin{pmatrix} 8 \\ -19 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 39 \\ 16 \end{pmatrix} \Rightarrow M_2(-6, 39, 16)$$

$$k_2: (x + 6)^2 + (y - 39)^2 + (z - 16)^2 = 1764$$

Aufgabe 10

(a) $M(1, -3, 0)$, $r = 5$, $P(1, 1, -3)$

$$\overrightarrow{MP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$$

$$4y - 3z + d = 0 \quad P(1, 1, -3) \in \tau$$

$$4 \cdot 1 - 3 \cdot (-3) + d = 0 \quad \Rightarrow \quad d = -13$$

$$\tau: 4y - 3z - 13 = 0$$

(b) $M(3, 0, 1)$, $r = 5$, $P(3, -4, -2)$

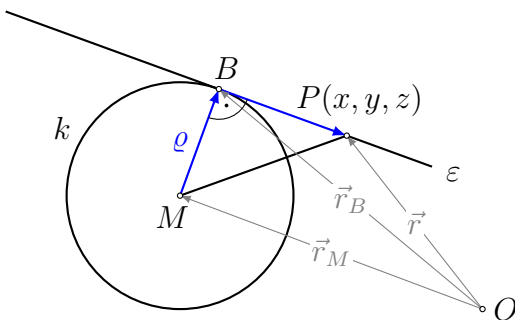
$$\overrightarrow{MP} = \begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ -3 \end{pmatrix} = - \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$$

$$4y + 3z + d = 0 \quad P(3, -4, -2) \in \tau$$

$$4 \cdot (-4) + 3 \cdot (-2) + d = 0 \quad \Rightarrow \quad d = 22$$

$$\tau: 4y + 3z + 22 = 0$$

Die Polare (Ebene durch $B \perp$ zu \overrightarrow{MB})



$$0 = \overrightarrow{MB} \cdot \overrightarrow{BP}$$

$$0 = (\vec{r}_B - \vec{r}_M) \cdot (\vec{r} - \vec{r}_B)$$

$$0 = (\vec{r}_B - \vec{r}_M) \cdot (\vec{r} - \vec{r}_M + \vec{r}_M - \vec{r}_B)$$

$$0 = (\vec{r}_B - \vec{r}_M) \cdot ((\vec{r} - \vec{r}_M) - (\vec{r}_B - \vec{r}_M))$$

$$0 = (\vec{r}_B - \vec{r}_M) \cdot (\vec{r} - \vec{r}_M) - (\vec{r}_B - \vec{r}_M) \cdot (\vec{r}_B - \vec{r}_M)$$

$$0 = (\vec{r}_B - \vec{r}_M) \cdot (\vec{r} - \vec{r}_M) - \rho^2 \quad \text{Polare von } k \text{ durch } B$$

Aufgabe 10 (Lösung mit der Polaren)

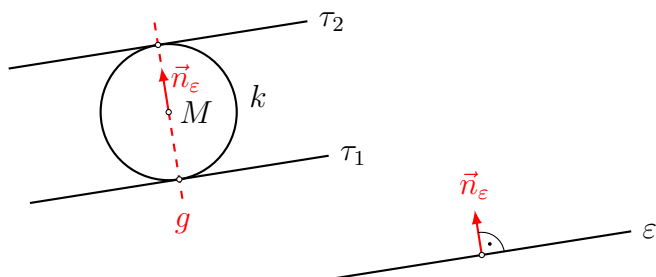
(a) $M(1, -3, 0)$, $r = 5$, $P(1, 1, -3)$

$$\begin{aligned}(\vec{r}_P - \vec{r}_M) \cdot (\vec{r} - \vec{r}_M) &= 0 \\ \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix} \cdot \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix} \right) - 25 &= 0 \\ 4y - 3z + 12 - 25 &= 0 \\ 4y - 3z - 13 &= 0\end{aligned}$$

(b) $M(3, 0, 1)$, $r = 5$, $P(3, -4, -2)$

$$\begin{aligned}(\vec{r}_P - \vec{r}_M) \cdot (\vec{r} - \vec{r}_M) &= 0 \\ \begin{pmatrix} 0 \\ -4 \\ -3 \end{pmatrix} \cdot \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right) - 25 &= 0 \\ -4y - 3z + 3 - 25 &= 0 \\ 4y + 3z + 22 &= 0\end{aligned}$$

Aufgabe 11



(a) $k: (x - 3)^2 + (y - 1)^2 + (z + 2)^2 = 49$, $\varepsilon: 3x + 2y - 6z = 0$

$$M(3, 1, -2); r = 7$$

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix}$$

$$g \cap k: 49 = (x - 3)^2 + (y - 1)^2 + (z + 2)^2$$

$$49 = (3t)^2 + (2t)^2 + (-6t)^2$$

$$49 = 49t^2$$

$$t_1 = 1 \Rightarrow B_1(6, 3, -8)$$

$$t_2 = -1 \Rightarrow B_2(0, -1, 4)$$

$$\tau_1: 3x + 2y - 6z - 72 = 0$$

$$\tau_2: 3x + 2y - 6z + 26 = 0$$

(b) $k: (x - 4)^2 + y^2 + (z + 1)^2 = 81$, $\varepsilon: 2x - 2y + z - 7 = 0$

$$M(4, 0, -1); r = 9$$

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$g \cap k: 81 = (x - 4)^2 + y^2 + (z + 1)^2$$

$$81 = (2t)^2 + (-2t)^2 + (t)^2$$

$$81 = 9t^2$$

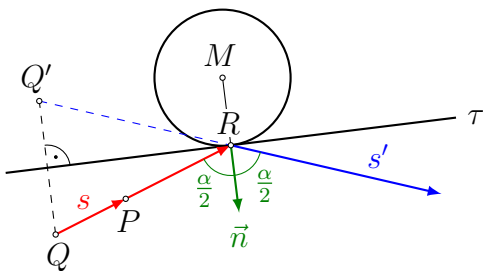
$$t_1 = 3 \Rightarrow B_1(10, -6, 2)$$

$$t_2 = -3 \Rightarrow B_2(-2, 6, -4)$$

$$\tau_1: 2x - 2y + z - 34 = 0$$

$$\tau_2: 2x - 2y + z + 20 = 0$$

Aufgabe 12



$$(a) \quad s: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 38 \\ -7 \end{pmatrix} + t \begin{pmatrix} -2 \\ -16 \\ 1 \end{pmatrix} \Rightarrow \begin{aligned} x &= -2t + 5 \\ y &= -16t + 38 \\ z &= t - 7 \end{aligned}$$

$s \cap k$:

$$225 = (-2t + 5 - 3)^2 + (-16t + 38 + 8)^2 + (t - 7)^2$$

$$225 = (-2t + 2)^2 + (-16t + 46)^2 + (t - 7)^2$$

$$0 = 261t^2 - 1494t + 1944$$

$$t_1 = \frac{108}{29} \approx 3.72 \quad \text{zu weit weg}$$

$$t_2 = 2 \Rightarrow R(1, 6, -5)$$

$$(b) \quad M(3, -8, 0), r = 15$$

$$\overrightarrow{MR} = \begin{pmatrix} 1 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 14 \\ -5 \end{pmatrix} = (-1) \cdot \begin{pmatrix} 2 \\ -14 \\ 5 \end{pmatrix}$$

$$d = -\vec{n} \cdot \vec{r}_R = 107 \Rightarrow \tau: 2x - 14y + 5z + 107 = 0$$

$$g: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 38 \\ -7 \end{pmatrix} + t \begin{pmatrix} -2 \\ 14 \\ 15 \end{pmatrix} \Rightarrow \begin{aligned} x &= -2t + 5 \\ y &= 14t + 38 \\ z &= -5t - 7 \end{aligned}$$

$$g \cap \tau: 0 = 2(-2t + 5) - 14(14t + 38) + 5(-5t - 7) + 107$$

$$0 = -225t - 450$$

$$t = -2 \Rightarrow t' = -4 \Rightarrow Q'(13, -18, 13)$$

$$\overrightarrow{QR} = \begin{pmatrix} 1 \\ 6 \\ -5 \end{pmatrix} - \begin{pmatrix} 13 \\ -18 \\ 13 \end{pmatrix} = \begin{pmatrix} -12 \\ 24 \\ -18 \end{pmatrix} = 6 \cdot \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$$

$$s': \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ -5 \end{pmatrix} + t \begin{pmatrix} -2 \\ 4 \\ -3 \end{pmatrix}$$

$$(c) \quad \alpha = 2 \arccos \frac{\vec{n} \cdot \vec{v}}{|\vec{n}| \cdot |\vec{v}|} = 2 \arccos \frac{75}{15 \cdot \sqrt{29}} = 43.60^\circ$$