

Aufgabe 1

(a) $\sqrt[3]{64} = 4$

(f) $\sqrt[17]{1} = 1$

(k) $\sqrt{576} = 24$

(b) $\sqrt[3]{27} = 3$

(g) $\sqrt[6]{729} = 3$

(l) $\sqrt[10]{1024} = 2$

(c) $\sqrt{441} = 21$

(h) $\sqrt[5]{32} = 2$

(m) $\sqrt[5]{1024} = 4$

(d) $\sqrt[5]{243} = 3$

(i) $\sqrt[7]{128} = 2$

(n) $\sqrt{1024} = 32$

(e) $\sqrt[3]{343} = 7$

(j) $\sqrt[3]{216} = 6$

(o) $\sqrt[8]{716} = 49$

Aufgabe 2

(a) $\sqrt[4]{\frac{1}{81}} = \frac{1}{3}$

(b) $\sqrt[6]{\frac{729}{256}} = \frac{3}{2}$

Aufgabe 3

(a) $\sqrt[5]{3\,200\,000} = \sqrt[5]{32 \cdot 10^5} = \sqrt[5]{32} \cdot \sqrt[5]{10^5} = 2 \cdot 10 = 20$

(b) $\sqrt[3]{2.16 \cdot 10^{11}} = \sqrt[3]{216 \cdot 10^9} = \sqrt[3]{216} \cdot \sqrt[3]{10^9} = 6 \cdot 10^3 = 6000$

Aufgabe 4

(a) $\sqrt[4]{0.00000016} = \sqrt[4]{16 \cdot 10^{-8}} = \sqrt[4]{16} \cdot \sqrt[4]{10^{-8}} = 2 \cdot 10^{-\frac{8}{4}} = 2 \cdot 10^{-2} = 0.02$

(b) $\sqrt[5]{0.00243} = \sqrt[5]{234 \cdot 10^{-5}} = \sqrt[5]{243} \cdot \sqrt[5]{10^{-5}} = 3 \cdot 10^{-1} = 3 \cdot 0.1 = 0.3$

Aufgabe 5

(a) $n < \sqrt{458} < n + 1$

$n^2 < 458 < (n + 1)^2$

$484 < 458 < 529$ (Potenzen auswendig kennen)

$22^2 < 458 < 23^2$

 $\sqrt{458}$ liegt zwischen 22 und 23.

(b) $n < \sqrt[3]{333} < n + 1$

$n^3 < 333 < (n + 1)^3$

$216 < 333 < 343$ (Potenzen auswendig kennen)

$6^3 < 333 < 7^3$

 $\sqrt[3]{333}$ liegt zwischen 6 und 7.

Aufgabe 10

(a) $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$

(b) $100^{\frac{3}{2}} = (10^2)^{\frac{3}{2}} = 10^3 = 1000$

(c) $32^{-\frac{2}{5}} = (2^5)^{-\frac{2}{5}} = 2^{-2} = \frac{1}{4}$

(d) $1024^{0.2} = (2^{10})^{\frac{1}{5}} = 2^2 = 4$

(e) $(\frac{1}{243})^{-\frac{3}{5}} = (3^{-5})^{-\frac{3}{5}} = 3^3 = 27$

(f) $0.25^{-\frac{1}{2}} = (\frac{1}{4})^{-\frac{1}{2}} = 4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^1 = 2$

Aufgabe 11

(a) $625 = 5^4$

(b) $\frac{1}{81} = \frac{1}{3^4} = 3^{-4}$

(c) $-216 = -6^3 = (-6)^3$

(d) $\frac{1}{\sqrt[3]{4}} = \frac{1}{4^{\frac{1}{3}}} = \frac{1}{2^{\frac{2}{3}}} = 2^{-\frac{2}{3}}$

Aufgabe 12

(a) $x^3 = 27$

$$x^3 = 3^3$$

$$x = 3$$

(b) $x^5 = -32$

$$x^5 = -2^5$$

$$x^5 = (-2)^5 \quad (\text{ungerader Exponent})$$

$$x = -2$$

(c) $x^6 = -64$

$$x^6 = -2^6$$

$$x^6 \neq (-2)^6 \quad (\text{gerader Exponent})$$

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(d) $x^4 = 256$

$$x^4 = (\pm 4)^4$$

$$x_{1,2} = \pm 4$$

Aufgabe 13

$$(a) \quad x^2 = \frac{1}{144}$$

$$x^2 = 12^{-2} = (12^{-1})^2$$

$$x_{1,2} = \pm 12^{-1} = \pm \frac{1}{12}$$

$$(b) \quad x^5 = \frac{1}{243}$$

$$x^5 = \frac{1}{3^5}$$

$$x^5 = 3^{-5} = (3^{-1})^5$$

$$x = 3^{-1} = \frac{1}{3}$$

$$(c) \quad x^3 = -\frac{27}{8}$$

$$x^3 = -\left(\frac{3}{2}\right)^3$$

$$x^3 = \left(-\frac{3}{2}\right)^3 \quad (\text{ungerader Exponent})$$

$$x = -\frac{3}{2}$$

$$(d) \quad x^4 = \frac{81}{10\,000}$$

$$x^4 = \left(\frac{3}{10}\right)^4$$

$$x = \pm \frac{3}{10} = 0.3$$

Aufgabe 14

$$(a) \quad x^3 = \sqrt{2}$$

$$x^3 = 2^{\frac{1}{2}} = (2^{\frac{1}{6}})^3$$

$$x = 2^{\frac{1}{6}} = \sqrt[6]{2}$$

$$(b) \quad x^4 = \sqrt[5]{16}$$

$$x^4 = (2^4)^{\frac{1}{5}}$$

$$x^4 = (2^{\frac{1}{5}})^4$$

$$x = \pm 2^{\frac{1}{5}} = \pm \sqrt[5]{2}$$

$$(c) \ x^5 = \sqrt[4]{7}$$

$$x^5 = 7^{\frac{1}{4}} = \left(7^{\frac{1}{20}}\right)^5$$

$$x = 7^{\frac{1}{20}} = \sqrt[20]{7}$$

$$(d) \ x^6 = \sqrt[3]{9}$$

$$x^6 = 3^{\frac{2}{3}} = 3^{\frac{6}{9}} = \left(3^{\frac{1}{9}}\right)^6$$

$$x = \pm 3^{\frac{1}{9}} = \sqrt[9]{3}$$

Aufgabe 15

$$(a) \ x^{3.5} = 128$$

$$x^{3.5} = 2^7 = 2^{2 \cdot 3.5} = (2^2)^{3.5}$$

$$x = 2^2 = 4$$

$$(b) \ x^{0.4} = \sqrt[5]{9}$$

$$x^{\frac{2}{5}} = \sqrt[5]{3^2} = 3^{\frac{2}{5}}$$

$$x = 3$$

$$(c) \ x^{\frac{4}{3}} = \frac{1}{16}$$

$$x^{\frac{4}{3}} = 2^{-4} = (2^{-3})^{\frac{4}{3}}$$

$$x = 2^{-3} = \frac{1}{8}$$

$$(d) \ x^{-\frac{2}{3}} = 25$$

$$x^{-\frac{2}{3}} = 5^2 = (5^{-3})^{-\frac{2}{3}}$$

$$x = 5^{-3} = \frac{1}{125}$$

Aufgabe 16

$$(a) \ x^{-4} = 16$$

$$x^{-4} = 2^4 = \left(\frac{1}{2}\right)^{-4}$$

$$x = \pm \frac{1}{2}$$

$$(b) \ x^{-3} = 343$$

$$x^{-3} = 7^3 = \left(\frac{1}{7}\right)^{-3}$$

$$x^{-3} = \frac{1}{7} = 7^{-1}$$

$$\begin{aligned} \text{(c)} \quad x^{-0.5} &= 3^{1.5} \\ x^{-0.5} &= (3^{-3})^{-0.5} \\ x &= 3^{-3} = \frac{1}{27} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad x^{-1.2} &= 8^{0.4} \\ x^{-1.2} &= (2^3)^{0.4} = (2^{-1})^{-1.2} \\ x &= 2^{-1} = \frac{1}{2} \end{aligned}$$

Aufgabe 17

$$\begin{aligned} \text{(a)} \quad 4^x &= 32 \\ 2^{2x} &= 2^5 \\ 2x &= 5 \\ x &= 2.5 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 27^x &= 9 \\ 3^{3x} &= 3^2 \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 25^x &= \frac{1}{5} \\ 5^{2x} &= 5^{-1} \\ 2x &= -1 \\ x &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 8^x &= \sqrt[3]{2} \\ 2^{3x} &= 2^{\frac{1}{3}} \\ 3x &= \frac{1}{3} \\ x &= 9 \end{aligned}$$

Aufgabe 18

$$\begin{aligned} \text{(a)} \quad 8^x &= 4^{33} \\ 2^{3x} &= 2^{66} \\ 3x &= 66 \\ x &= 22 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 1000^x &= 100^{-45} \\
 10^{3x} &= 10^{-90} \\
 3x &= -90 \\
 x &= -30
 \end{aligned}$$

Aufgabe 19

$$\begin{aligned}
 \text{(a)} \quad 7^{4x} &= \sqrt[5]{49} \\
 7^{4x} &= 49^{\frac{1}{5}} = 7^{\frac{2}{5}} \\
 4x &= \frac{2}{5} \\
 x &= \frac{1}{10}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 9^x &= \frac{1}{\sqrt{27}} \\
 3^{2x} &= \frac{1}{3^{\frac{3}{2}}} \\
 3^{2x} &= 3^{-\frac{3}{2}} \\
 2x &= -\frac{3}{2} \\
 x &= -\frac{3}{4}
 \end{aligned}$$

Aufgabe 20

$$\begin{aligned}
 \text{(a)} \quad 6^{7x} &= 1 \\
 6^{7x} &= 6^0 \\
 7x &= 0 \\
 x &= 0 \\
 \\
 \text{(b)} \quad 9^{x+5} &= 27^{x-4} \\
 3^{2(x+5)} &= 3^{3(x-4)} \\
 2x + 10 &= 3x - 12 \\
 x &= 22
 \end{aligned}$$

Aufgabe 21

$$\begin{aligned}
 \text{(a)} \quad 0.1^x &= 10^2 \\
 \left(\frac{1}{10}\right)^x &= 10^2 \\
 10^{-x} &= 10^2 \\
 -x &= 2 \\
 x &= -2
 \end{aligned}$$

$$\begin{aligned}
\text{(b)} \quad & \frac{1}{2} \cdot 2^x = 8 \cdot \frac{1}{4^x} \\
& 2^{-1} \cdot 2^x = 2^3 \cdot 2^{-2x} \\
& 2^{-1+x} = 2^{3-2x} \\
& -1 + x = 3 - 2x \\
& 3x = 4 \\
& x = \frac{4}{3}
\end{aligned}$$

Aufgabe 22

$$\begin{aligned}
\text{(a)} \quad & 4^x + 16 = 10 \cdot 2^x \\
& (2^x)^2 - 10 \cdot 2^x + 16 = 0 \quad \text{Substitution: } 2^x = a \\
& a^2 - 10a + 16 = 0 \\
& (a - 2)(a - 8) = 0 \\
& a_1 = 2 = 2^1 = 2^x \quad \Rightarrow \quad x_1 = 1 \\
& a_2 = 8 = 2^3 = 2^x \quad \Rightarrow \quad x_2 = 3
\end{aligned}$$

$$\begin{aligned}
\text{(b)} \quad & 9^x - 8 \cdot 3^x = 9 \\
& (3^2)^x - 8 \cdot 3^x = 9 \\
& (3^x)^2 - 8 \cdot 3^x - 9 = 0 \quad \text{Substitution: } 3^x = a \\
& a^2 - 8a - 9 = 0 \\
& (a - 9)(a + 1) = 0 \\
& a_1 = 9 = 3^2 = 3^x \quad \Rightarrow \quad x_1 = 2 \\
& a_2 = -1 = 3^x \quad \Rightarrow \quad \text{keine Lösung}
\end{aligned}$$