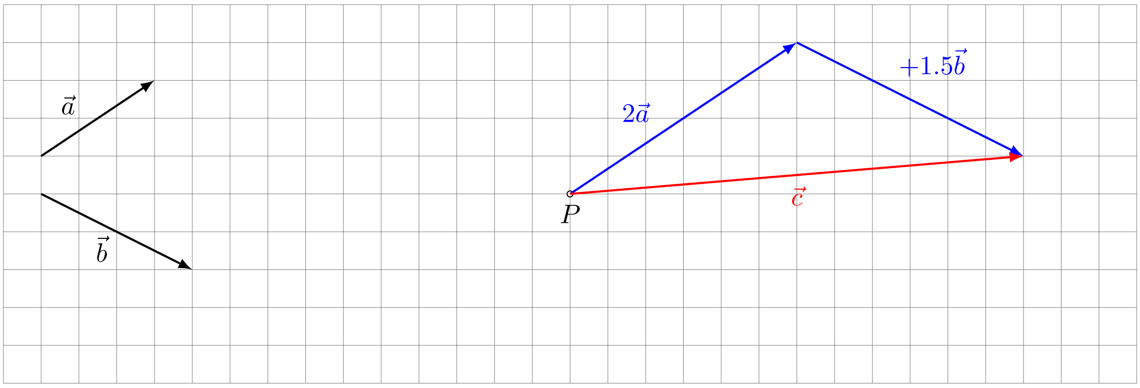
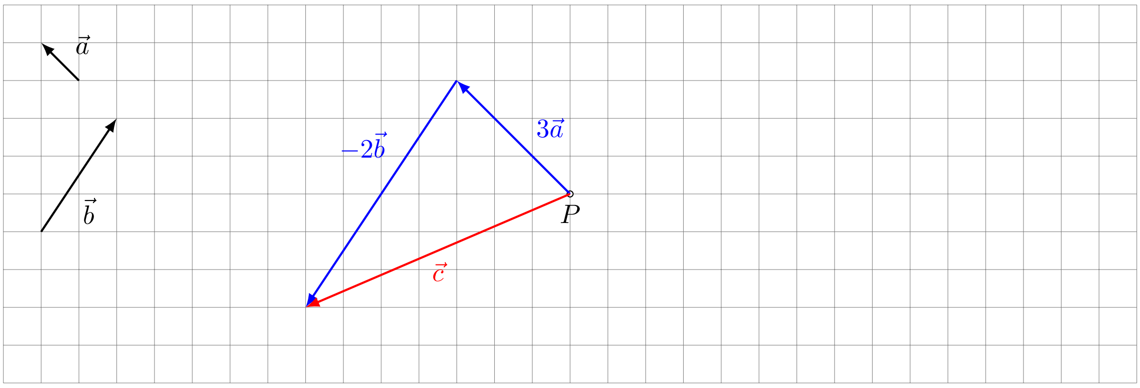
Kompetenztest 3 **Lösungen+** **Übungstest**

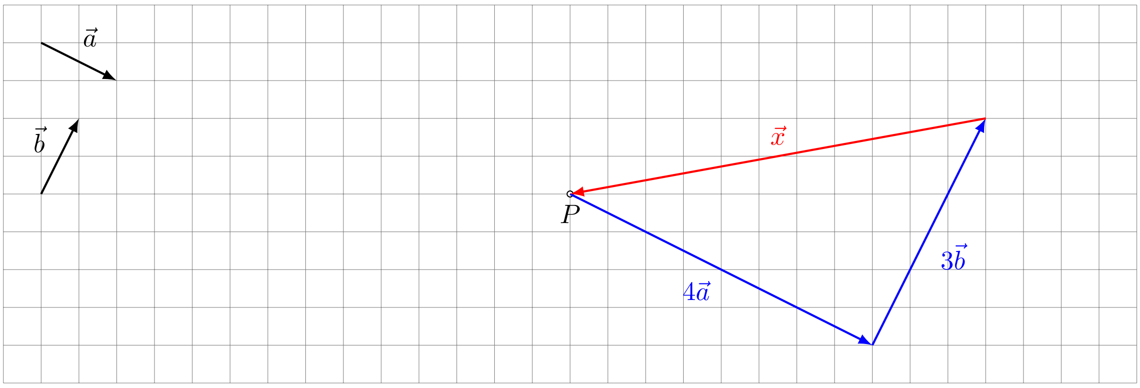
1. $\vec{c} = 2\vec{a} + 1.5\vec{b}$



2. $\vec{c} = 3\vec{a} - 2\vec{b}$



3. $4\vec{a} + 3\vec{b} + \vec{x} = \vec{0}$



4. $2\vec{x} - 3\vec{a} + \vec{b} = \vec{a} - \frac{1}{2}\left(\vec{x} + \vec{b}\right) \quad || \cdot 2$

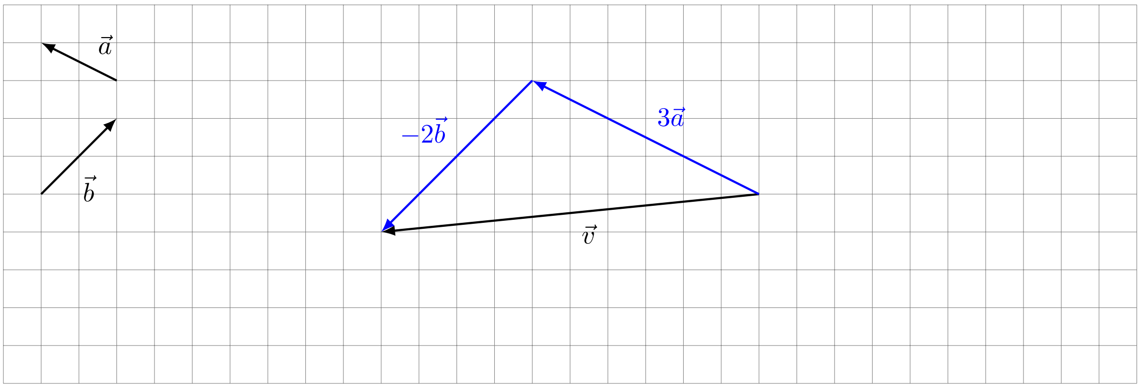
$$4\vec{x} - 6\vec{a} + 2\vec{b} = 2\vec{a} - \left(\vec{x} + \vec{b}\right)$$

$$4\vec{x} - 6\vec{a} + 2\vec{b} = 2\vec{a} - \vec{x} - \vec{b}$$

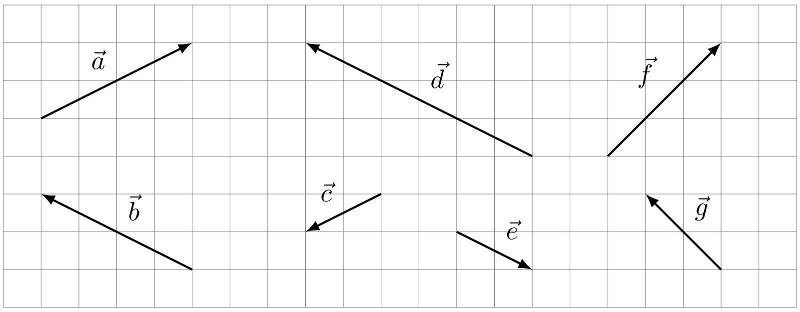
$$5\vec{x} = 8\vec{a} - 3\vec{b}$$

$$\vec{x} = \frac{8}{5}\vec{a} - \frac{3}{5}\vec{b}$$

5. $\vec{v} = 3\vec{a} - 2\vec{b}$



6.
- $\vec{a}$ und $\vec{c}$
- $\vec{b}$, $\vec{d}$ und $\vec{e}$

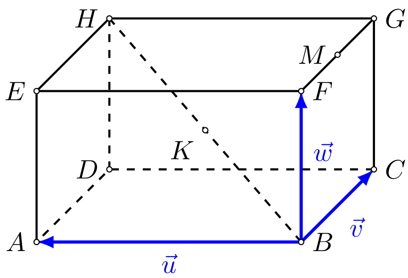


7. $\vec{v} = \vec{a} + \frac{1}{2}\vec{b} - 3\vec{c} = \begin{pmatrix} 7 \\ -1 \\ 4 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix} - 3\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$

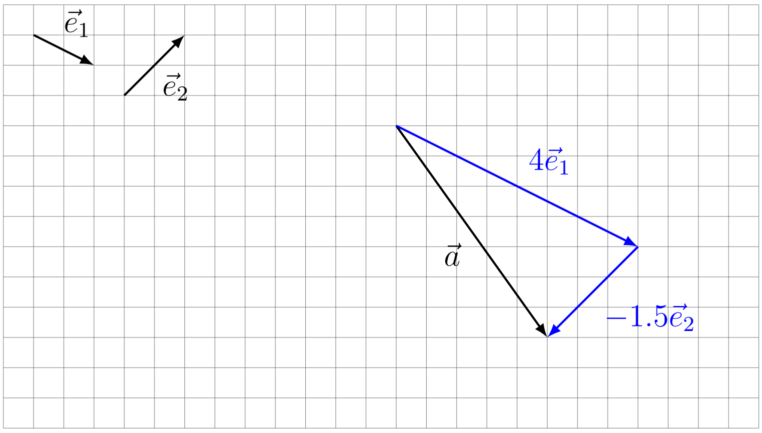
8. (a) $\overrightarrow{GA} = \vec{u} - \vec{v} - \vec{w}$

(b) $\overrightarrow{DM} = -\vec{u} - \frac{1}{2}\vec{v} + \vec{w}$

(c) $\overrightarrow{EK} = -\frac{1}{2}\vec{u} + \frac{1}{2}\vec{v} - \frac{1}{2}\vec{w}$



9. $\vec{a} = \begin{pmatrix} 4 \\ -1.5 \end{pmatrix}$



10. $\begin{pmatrix} a \\ 2 \\ 5 \end{pmatrix} + 3\begin{pmatrix} -1 \\ b \\ 4 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 3 \\ c \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix}$

$$\begin{aligned} a - 3 + 0 &= 7 \\ 2 + 3b - 6 &= -1 \\ 5 + 12 - 2c &= 3 \end{aligned} \quad \Rightarrow \quad \begin{aligned} a &= 10 \\ 3b &= 3 \\ -2c &= -14 \end{aligned} \quad \Rightarrow \quad \begin{aligned} a &= 10 \\ b &= 1 \\ c &= 7 \end{aligned}$$

11. Zwei Vektoren sind kollinear, wenn es eine Zahl k gibt, so dass $\vec{a} = k\vec{b}$ (oder $k\vec{a} = \vec{b}$).

$$\begin{pmatrix} 12 \\ -8 \\ 20 \end{pmatrix} = k\begin{pmatrix} -9 \\ 6 \\ -12 \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} 12 &= -9k \\ -8 &= 6k \\ 20 &= -12k \end{aligned} \quad \Rightarrow \quad \begin{aligned} k &= -\tfrac{4}{3} \\ k &= -\tfrac{4}{3} \\ k &= -\tfrac{5}{3} \end{aligned}$$

Das „Gleichungssystem“ hat keine Lösung $\Rightarrow$ $\vec{a}$ und $\vec{b}$ sind nicht kollinear.

12. Löse die Gleichung $x\vec{a} + y\vec{b} + z\vec{c} = \vec{v}$:

$$x\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + y\begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + z\begin{pmatrix} -3 \\ 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \\ 7 \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} 2x + y - 3z &= -1 \\ -x + 3y + 6z &= 6 \\ 5x + 4y - 2z &= 7 \end{aligned} \quad \overset{\text{TI 30X}}{\Rightarrow} \quad \begin{aligned} x &= 3 \\ y &= -1 \\ z &= 2 \end{aligned}$$

Also gilt $\vec{v} = 3\vec{a} - \vec{b} + 2\vec{c}$.

13. $|\vec{v}| = \sqrt{3^2 + (-14)^2 + 18^2} = \sqrt{9 + 196 + 324} = \sqrt{529} = 23$

14. $\overrightarrow{AB} = \vec{r}_B - \vec{r}_A = \begin{pmatrix} 5 \\ 7 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \\ 9 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ -8 \end{pmatrix}$

$\text{dist}(A, B) = |\overrightarrow{AB}| = \sqrt{4^2 + 1^2 + (-8)^2} = \sqrt{16 + 1 + 64} = \sqrt{81} = 9$

15.
$$\begin{aligned} p &= \frac{F}{A} \\ Ap &= F \\ A &= \frac{F}{p} \end{aligned}$$

16. $$\begin{aligned} v &= v_0 + at \\ at &= v - v_0 \\ a &= \frac{v - v_0}{t} \end{aligned}$$

17. $$\begin{aligned} r_1F_1 &= (r_2 - x)F_2 + xF_3 \\ r_1F_1 &= r_2F_2 - xF_2 + xF_3 \\ r_1F_1 - r_2F_2 &= x(-F_2 + F_3) \\ x &= \frac{r_1F_1 - r_2F_2}{F_3 - F_2} \end{aligned}$$

18. (a) $\dfrac{1}{243} = \dfrac{1}{3^5} = 3^{-5}$

(b) $\sqrt[5]{49} = 49^{\frac{1}{5}} = (7^2)^{\frac{1}{5}} = 7^{\frac{2}{5}}$

19. $$\begin{aligned} x^{30} &= 8^{1000} \\ x^{30} &= (2^3)^{1000} \\ x^{30} &= (2^{1000})^3 \\ x^{30} &= (2^{100})^{30} \\ x &= \pm 2^{100} \end{aligned}$$

20. $$\begin{aligned} 8^{2x-7} &= 4^{2x-5} \\ (2^3)^{2x-7} &= (2^2)^{2x-5} \\ 2^{3(2x-7)} &= 2^{2(2x-5)} \\ 2^{6x-21} &= 2^{4x-10} \\ 6x - 21 &= 4x - 10 \\ 2x &= 11 \\ x &= \tfrac{11}{2} \end{aligned}$$