Kompetenztest 2 **Lösungen+** **Übungsversion**

1. $d = \sqrt{a^2+b^2+c^2} = \sqrt{4+36+9} = 7\,\text{cm}$

2. $S = 2G + M \quad = 2 \cdot \dfrac{\sqrt{3}}{4} \cdot 4 + 3 \cdot 2 \cdot 5 = 33.46\,\text{cm}^2$

3. $V = \frac{1}{3} \cdot G \cdot h = \dfrac{1}{3} \cdot \pi \cdot r^2 \cdot h = \dfrac{1}{3} \cdot \pi \cdot 3^2 \cdot 8 = 24\pi\,\text{cm}^3 = 75.40\,\text{cm}^3$

4. $V = \dfrac{4}{3}\pi r^3 \quad \Rightarrow \quad r^3 = \dfrac{3V}{4\pi} \quad \Rightarrow \quad r = \sqrt[3]{\dfrac{3V}{4\pi}} = 2.12\,\text{cm}$

5. ☒ Ein lineares Gleichungssystem kann mehr als eine Lösung haben.
   ☐ Die Lösungsmenge einer linearen Gleichung verändert sich, wenn man beide Seiten dieser Gleichung mit einer Zahl $a \neq 0$ multipliziert.
   ☒ Das Zahlenpaar $(-2, 1)$ ist eine Lösung der Gleichung $-x + 3y = 5$.

6. (a) $3x - 2y = 5 \quad [1]$ $\qquad 2 \cdot [1] + [2]$: $13x + 0 = 26$
   $7x + 4y = 16 \quad [2]$

$$x = 2$$
$$6 - 2y = 5$$
$$y = \tfrac{1}{2}$$
$$L = \{(2, \tfrac{1}{2})\}$$

   (b) $3(x+y) + 2y = 7 \quad [1]$ $\qquad (x+y) = 6 + 3y$ in [1]: $\quad 3(6+3y) + 2y = 7$
   $(x+y) - 3y = 6 \quad [2]$

$$18 + 9y + 2y = 7$$
$$11y = -11$$
$$y = -1$$
$$(x-1) = 6 - 3$$
$$x = 4$$
$$L = \{(4, -1)\}$$

   (c) $2x - 3y = 5 \quad [1]$ $\qquad [1] + [2]$: $\quad 0 = 8$
   $-2x + 3y = 3 \quad [2]$ $\qquad L = \{\,\}$

7. $2x - 2y + 3z = 4 \qquad 3z = -6 \qquad 5y - 2 = 13 \qquad 2x - 6 - 6 = 4$

$$\begin{array}{rcccc} 3z = -6 & & z = -2 & 5y = 15 & 2x = 16 \\ 5y + z = 13 & & & y = 3 & x = 8 \end{array}$$

$L = \{(8, 3, -2)\}$

8. (a) $x^2 - 10x = 0 \quad \Rightarrow \quad x(x - 10) = 0 \quad \Rightarrow \quad L = \{0, 10\}$
   (b) $x^2 + 9 = 0 \quad \Rightarrow \quad L = \{\,\}$
   (c) $9x^2 - 4 = 0 \quad \Rightarrow \quad 9x^2 = 4 \quad \Rightarrow \quad x^2 = \frac{4}{9} \quad \Rightarrow \quad L = \{\pm\frac{2}{3}\}$
   (d) $x^2 + 2x - 15 = 0 \quad \Rightarrow \quad (x + 5)(x - 3) = 0 \quad \Rightarrow \quad L = \{-5, 3\}$

9. $2x^2 - 5x + 1 = 0$

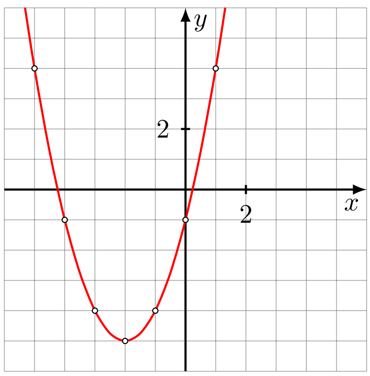
$$D = \sqrt{b^2 - 4ac} = \sqrt{(-5)^2 - 4 \cdot 2 \cdot 1} = \sqrt{17}$$

$$x_1 = \frac{-b + \sqrt{D}}{2a} = \frac{5 + \sqrt{17}}{4}$$

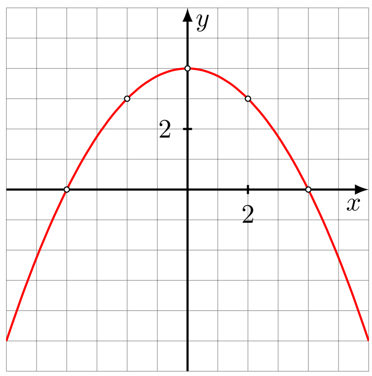
$$x_2 = \frac{-b - \sqrt{D}}{2a} = \frac{5 - \sqrt{17}}{4}$$

10. (a) $f(x) = (x - 3)^2 + 2 \quad \Rightarrow \quad S(3, 2)$
    (b) $f(x) = x^2 + 4x + 5 = x^2 + 4x + 4 - 4 + 5 = (x + 2)^2 + 1 \quad \Rightarrow \quad S(-2, 1)$
    [oder mit den Formeln zur Berechnung der Scheitelpunktkoordinaten]

11. (a) $f(x) = (x + 2)^2 - 5$



(b) $f(x) = -\frac{1}{4}x^2 + 4$



12.
$$\begin{aligned} x^2 - 2x + 6 &= 2x + 3 \\ x^2 - 4x + 3 &= 0 \\ (x-1)(x-3) &= 0 \end{aligned}$$
$$x_1 = 1 \quad \Rightarrow \quad y_1 = 5 \quad \Rightarrow \quad P_1(1,5)$$
$$x_2 = 3 \quad \Rightarrow \quad y_2 = 9 \quad \Rightarrow \quad P_2(3,9)$$

13. (a) $a^{m+3} \cdot a^{m-3} = a^{2m}$

(b) $(14c)^5 : (7c)^5 = (14c : 7c)^5 = 2^5 = 32$

(c) $\left(\frac{v}{w}\right)^n \cdot \left(\frac{w}{v}\right)^n = \left(\frac{v}{w} \cdot \frac{w}{v}\right)^n = 1^n = 1$

(d) $x : x^{n-5} = x^{1-(n-5)} = x^{6-n}$

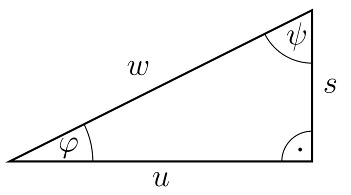
(e) $\dfrac{x^7 + x^5}{x^5 + x^3} = \dfrac{x^5(x^2+1)}{x^3(x^2+1)} = x^2$

(f) $\left(\dfrac{a^5}{b^2}\right)^{-3} \cdot \left(\dfrac{b^3}{a}\right)^7 = \left(a^5 b^{-2}\right)^{-3} \cdot \left(b^3 a^{-1}\right)^7 = a^{-15} b^6 b^{21} a^{-7} = a^{-22} b^{27}$

14. (a)
$$\begin{aligned} x^4 &= 16 \\ x^4 &= (\pm 2)^4 \\ x &= \pm 2 \end{aligned}$$

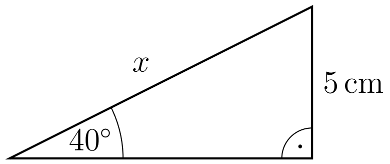
(b)
$$\begin{aligned} 5^{3x+1} : 5^{x-2} &= 5^{x+7} \\ 5^{3x+1-(x-2)} &= 5^{x+7} \\ 2x + 3 &= x + 7 \\ x &= 4 \end{aligned}$$

15.



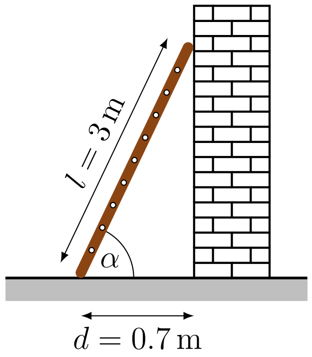
- ☐ w ist die Ankathete von φ.
- ☒ u ist die Gegenkathete von ψ.
- ☒ $\cos(\varphi) = \dfrac{u}{w}$
- ☒ $\psi = \arctan\left(\dfrac{u}{s}\right)$

16.



$$\sin(40°) = \frac{5}{x} \quad \Rightarrow \quad x = \frac{5}{\sin(40°)} = 7.78\,\text{cm}$$

17.



$$\cos(\alpha) = \frac{0.7}{3} \quad \Rightarrow \quad \alpha = \arccos\frac{0.7}{3} = 76.51°$$